G.A. STEM

Enhancing STEM skills through Arts and mini-games

O1 - FRAMEWORK TO INTEGRATE ART IN STEM USING DIGITAL GAMES

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Activity related	O1/A1 - Analysing the "mathematization" concept in relation to STEM schedule/exercises using the arts O1/A2 - Analysing of national school curriculum for math and science of 13-16 years old students O1/A3 - Identifying the art works that better represent "connections" with maths and science O1/A4 - Analysing the existing practices of the use of the games for math and science study for 13-16 years old students O1/A5 - Analysing game and development elements
Deliverable N° and title	**IO1 - Framework to integrate Art in STEM using digital games**

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1. Introduction

1.1 Background and overview of the report

The worldwide surveys, such as Program for International Student Assessment (PISA) and Trends in International Mathematics and Science Study (TIMSS), show that European students often lack mathematical competences and key basic competences in science and technology. Actually, their achieved performance results in mathematics are below the OECD (Organisation for Economic Co-operation and Development) average (OECD 2016; Eurydice 2017).

Even if some European countries, like Finland, are the highest performing OECD countries, the most recent PISA results (2015) have shown a drop of the Finnish averages in scientific skills. Actually, during an interview for "The Guardian" Newspaper published on 07/01/2018, Pasi Sahlberg, key actor in Finnish education policy, has noted that the increasing of digital technologies use for communication, interaction and entertainment will complicate the focusing on complex conceptual issues, such as those met in mathematics and science.

Another problem is that Finland is the only country where girls are more likely to be top performers than boys, in contrast to other European countries, in which girls often are not so encouraged to pursue a scientific career. This determines major regional disparities.

Estonian students, for their part, have obtained a score in science 534 on OECD average of 493 (+2), in mathematics 520 on OECD average of 490 (+2).

Thanks to their digital development and education quality, they achieved both high levels of performance and a greater equity in education outcomes. The most critical problems of the Estonian education system, instead, are related to teachers. A recent review by the OECD (2016) identified some policy priorities to improve the effectiveness of the Estonian school system: consolidate school networks, promote professionalism of teachers and school leaders and turn vocational education into a more attractive option.

In other countries, as Italy and Belgium, the scores are above or on OECD average. Negative trends are emerged by Belgian students who obtained a score in science 502 on OECD average of 493 (-3), in mathematics 507 on OECD average of 490 (-5).

In Italy, students obtained a score in science 481 on OECD average of 493 (+2); in mathematics 490 on OECD average of 490 (+7). The scores are getting better, but the changing is very slow and difficult. The Report on PISA 2015 results has revealed that more time spent during the after school studying is not enough, and students need more time in regular lessons with better teaching as well.

This situation has been demonstrated also by the decreasing interest and motivation of young people in starting up the ICT and science careers.
As Schreiner and Sjøberg suggest that "it might be that we have now passed the era in which the work of physicists, technicians and engineers is seen as crucial to people's lives and well-being". Today's youth is more interested in who they will be rather than what they will do. Negative stereotypes of scientists, engineers, researchers and other STEM (science, technology, engineering and math) experts' career can be found among European youth. There is a lack of attractive role models and a lack of information and understanding of what careers in STEM are about.

Another issue related to STEM is gender differences. Girls are often not being encouraged to pursue a scientific career. There is an insufficient number of girls taking up or being encouraged to take up these subjects. Too often teachers and career advisers still call on more traditional thinking that STEM is more a male domain. In this context it will be important to create new instruments in order to train and prepare people with the right skill sets for the period when new jobs will come on stream.

In this context, the specific common national needs are emerged among the countries involved:

1. Emphasizing the joy of learning - According to Finnish system, the best way to tackle the problem of insufficient school results is not to raise standards or increase time spent teaching (or homework), but make school a more interesting and fun place for everyone.
2. Improving social inclusion and gender equity in education.
3. Developing schools as learning communities - Favouring a more interdisciplinary and multidisciplinary teaching and learning in European schools, which will lead to a better collaboration among teachers from different subjects.
4. Reinforcing school and teachers networks to share resources and best practices. Sense of community and networking are key elements for the development and modernisation of teacher training.

Therefore, G.A.STEM project responds to these needs with the following objectives:

1. Improving motivation in scientific study through the use of "Art-works" as a supporter in student creativity development and raising awareness of their applications in everyday life.
2. Utilizing the attractiveness of the art and technology (in terms of mini-games design and game assets) to improve social inclusion and gender equality.
3. Supporting STEM skills (vertical and horizontal skills) useful for professional careers for both teachers and students.

4. Improving the collaborative sense among teachers and schools through the exchanging of experience, best practices focusing on the interdisciplinary and multidisciplinary approach.
5. Increasing the community sense and citizenship awareness through the discovery of European Cultural heritage constituted of(past and present) art-works produced in the partner project countries.

1.2 Purpose and research questions of the report

The purpose of this report is to offer a framework focused on how to integrate Art in STEM (science, technology, engineering and mathematics) education using the mini-games design and setting game scenarios.

The research questions are:

- What are the special features/components of STEAM? and
- How STEAM practices can be developed for middle school classrooms in the context of using digital games?

The report will describe the criteria used by RET (Research and Evaluation Team) in order to select the exercises on math and science to be used for the realization of the study projects with the secondary school students during the piloting phase in the G.A. STEM project. It will also contain all the selected exercises which all together means at least 10 exercises - 4 from international surveys and 6 from national surveys; two from each partner country involved - for 13-16 year old students.

1.3 Structure and progress of the report

The report is structured in such a way that the theoretical part of the framework is presented at the beginning of the report. The theoretical part is followed by the empirical part which describes the criteria and implementing of the project activities in IO1. In the report the following G.A. STEM project activities are described and considered as crucial in constructing the framework on how to integrate art in STEM using digital games:

- O1/A1 - Analysing the “mathematization” concept in relation to STEM schedule/exercises using the arts;
- O1/A2 - Analysis of national school curriculum for math and science of 13-16 years old students (Grade 8 – 12 -13 years old and Grade 9 – 13-14 years old)

- O1/A3 - Identifying the art works that better represent “connections” with maths and science
- O1/A4 - Analysing the existing practices of the use of the games for math and science study for 13-16 years old students
- O1/A5 - Analysing game design and development elements.

2. Analysing the "mathematization" concept in relation to STEM schedule/exercises using the arts

Using the arts in STEM, or so to say, the STEAM approach, leads to an analysis of the concept of "mathematization". In this chapter, besides the analysis of the concept of "mathematization", the STEAM approach with the questions of why and how to combine the STEM subjects with arts and bring scientific problems into an artistic context will be examined. In this report the combination with STEM and art will be described through suggestions, examples and experiences of the existing examples of STEAM practices and real-life applications, the focus being specifically on those concerning secondary school.

2.1 From STEM to STEAM approach

The acronym STEM was originally created at 2001 by Dr. Judith Ramaley, assistant director of the Education and Human Resources Directorate, at NSF (Chute, 2009) and defined as an educational inquiry where learning is contextual, based on real-world problems and contained innovation activity. (Daugherty 2013).

According to Wynn and Harris (2012) the acronym, STEM which includes subjects of science, technology, engineering, and mathematics, has evolved into STEAM, where "A" stands for art. The approach is based on the new opportunities that arts bring into learning and studying. Combining art into STEM leads to "a different way of perceiving the world", which offers an opportunity to "expand the toolbox" and "free the scientist's and engineer's mind" (Storksdieck 2011).

2.2 Why to combine STEM with arts

Hunter-Doniger and Sydow (2016) point out that the connections through STEM and the arts help in attracting the students' interest, engagement, innovation and creativity in solving the scientific problems related to STEM subjects.

According to Wynn and Harris (2012), in the classroom and in learning, the integration of STEM and art, STEAM, has been noticed to produce creative thinking and 21st century skills (Maeda 2012), stimulation and inspiration, engagement, and innovation as well as ingenuity, in solving real-world related problems. This connection with the reality makes science more interesting to growing children, especially to those who are right-brain oriented as learners. (Wynn & Harris 2012).

The OECD recommendations as a result of the PISA tests indicate U.S. students' satisfaction and positive attitude towards school and teacher relationships, but lack of strong motivation towards learning mathematics. Only around half of the students, both boys and girls, agreed or strongly agreed about being interested in learning mathematics. This is worrying especially because in today's world, motivation is one of the qualities sought by employers. The motivation comes to the students of this generation through connections to the real world applications. (De Simone 2014).

As it stands in the OECD, report the students need to focus more on higher-order activities in their learning, however without neglecting the basic skills needed to be able do that in the background. The students need the basic core subjects of STEM to have the tools to work with, then they need another contexts to explore, learn and create. The STEAM approach is whole brain thinking based approach that shows that subjects are not isolated from each other. (De Simone 2014).

According to Fogarty, Winey, Howe, Hancox and Whyley (2016) many of today's large problems such as hunger and climate change need broader, multidisciplinary, views and solutions that are related to wider approach which doesn't rely solely on STEM, as important and obvious as the STEM aspects are. The arts combination is needed for the more balanced education of the students. (Fogarty et al. 2016).

Today's economy needs different thinkers who have originality, flexibility and skills for innovation (Moses and Cobb 2001). The arts create 21st century skills (such as creativity, problem solving, critical thinking, collaboration) needed by students who will become adults of today's digital world (Sousa & Pilecki 2013). Since we cannot yet know what might the career paths of the future be like, students need to be multidisciplinary and able to combine different disciplines (Madden et al. 2013; Riley 2012; Hetland 2013). The STEAM approach can provide an answer for this by connecting traditional STEM disciplines into creative art subjects (Watson and Watson 2013). (Hunter-Doniger and Sydow 2016).

According to Quigley and Herro (2016) STEAM brings the learning process and problem-solving authenticity and new approaches by combining into the activities arts that naturally deliver interdisciplinary inquiry. Even if the approach is quite new there are already some findings indicating that motivation, engagement and effectiveness in disciplinary learning have increased in STEAM as well as in STEM education (Henrkisen et al. 2015, Davis 2014). According to the research also students' ability to make connections between the disciplines seems to have increased (Connor et al.

2015). Generally, the transdisciplinary STEAM approach can be characterized with open and creative thinking and use of imagination, which leads to innovation (Mishra et al. 2011, Eisner 2002). (Quigley and Herro 2016).

According to English (2017) both STEM and STEAM learning experiences, when well designed, have a potential to provide increased participation, engagement, motivation and achievement among students.

2.3 How to combine STEM with arts

The idea of how arts should be combined with STEM subjects is not easily defined. According to Colucci-Gray, Burnard, Cooke, Davies, Gray and Trowsdale (2017) both STEM and the arts are ambiguous already as concepts and the word STEAM is often used differently depending on context. There remains little theorizing about the Arts and STEM combinations that have worked effectively and achieved the expected educational and economic outcomes. (Colucci-Gray et al. 2017).

According to De Simone (2014) the arts are often fit into other curricula rather than standing on their own. However, when they stand on their own, they automatically fit into other subjects: math's ideas and logic complement the arts, music theory is based on mathematical principles, and art is involved with geometry. Similarly, the study of the history of the arts can be applied to the study of civilization. Art and music have had their place in every great society, from primitive to highly-advanced. Students who learn about the great art and music from any historical period will gain an appreciation for those who lived during those times. (DeSimone 2014).

Critics of the STEAM movement suggest that combining arts and STEM together either decreases the emphasis on science or weakens the aesthetic experience (Jolly 2014). However the relationship between STEM subjects and arts shouldn't be seen an either–or proposition but rather as a spiral that bends back upon itself to revisit complex ideas in depth. Students are likely to gain more with arts and sciences, expanding their interests in both subjects (Toro 2014; Tzou et al. 2014). (Simpson Steele, Fulton & Fanning 2016).

In the following are a few examples of how STEAM practices have been implemented in the middle school in recent years:

In 6th grade ELA (=English Language Arts) class in the town of Mount Pleasant, South Carolina, there was being used a narrative writing assignment in which the students created their own picture books by using technology such as digital tools to make pop-ups, animated features and other effects. A group of students was also trained to teach others as students' own learning lab activity. Technology was used with a purpose which made learning it more intense and meaningful. As a result of the project it could be seen that the use of technology enhanced students to become more engaged to the writing process itself. All in all the Whole-School's STEM program helped students to understand that the subjects are complementary and connected together. It also deepened the learning process and ignited the enthusiasm, capabilities and success of some students who had not previously been interested in school. (Goodwin et al. 2016, Goodwin et al. 2014)

A 5th grade teacher in Kahulu, Hawaii, used STEM and arts integration through dance when exploring energy transforming, focused on the properties of wind and how to store, transform and use its energy. The dance was integrated into the process at different stages to help her students learn the science concepts while applying choreographic principles. According to the results the students combined multiple learning experiences to understand complex phenomena and expressed them visually, kinesthetically, verbally and manually. (Simpson Steele, Fulton and Fanning 2016)

In Atlanta's STEAM experiment electronic puppet-building workshops were designed for middle to early high school students in order to combine electronics with traditional puppet building workshops, and to teach middle school to early high school students the basic electronic circuit building in this setting. (Peer, Nitsche & Schaffer 2014).

In Virginia middle school's after school program called the Linux Laptop Orchestra (L2Ork) the target group, at-risk students, combined programming with art by creating music by using some basic computer programming components and small laptops. As a result of the program students learned basic programming skills in a relatively short time, their technological literacy increased and they showed different learning preferences in different situations. Even if they didn't come experts in either music or computer science, they built interest in both subjects and learnt new skills and terminology in each discipline. (Moyer, Klopfer & Ernst 2018).

2.4 Mathematization

In Figure 1 it is described how a real world problem is transformed into a mathematical form, solved, and validated the solution against the original context of the real problem. The original model is taken from a US policy document on standards in mathematics (NCTM, 1989), and adopted by PISA in a little transformed version (OECD, 2003, p. 38). On the right side of Figure 1 lays the mathematical and the abstract world, with its well-defined symbols and rules. On the left side is the real concrete world; daily life. Working with pure mathematics, such as numbers or algebra without a context, the process takes place on the right side of Figure 1. In applied mathematics, the process takes place on the left side; in the real world. In this case, the real problem has to be simplified at first, and then mathematized into a model representing it. In school mathematics a real problem is rarely the starting point; usually the problem presented to the students has already been simplified. (Grønmo and Olsen, 2009).

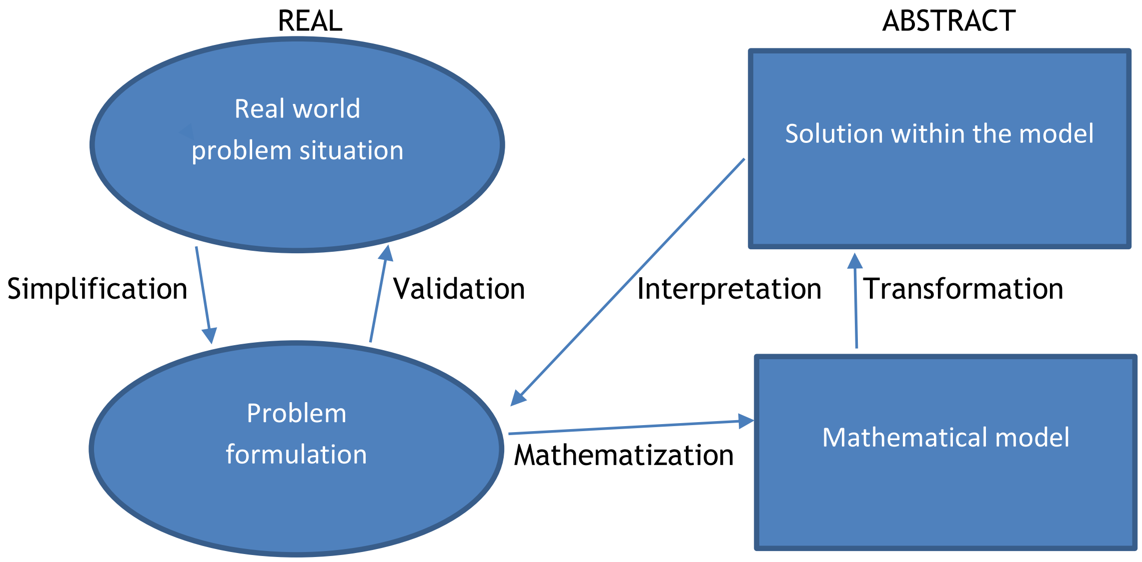

Figure 1: The mathematization cycle. Copied from the Standards (NCTM, 1989). (Grønmo and Olsen, 2009).

Based on the mathematization cycle in Figure 1 it is obvious that before being able to work with any type of applied mathematics the students must have the necessary knowledge in pure mathematics to solve any problems correctly. The students must have skills that they can orientate themselves in a pure mathematics' world. This demonstrates that even if the mathematical process needed is the same in both pure and applied mathematics, applied mathematics is however more complex in any case. Some countries' increased focus on applied mathematics has been criticized (i.e. Gardiner 2014) by saying that at the expense of trying to enhance the mathematical literacy, the orientation into pure mathematics and the importance of mathematics as an exact and well-defined science as such is neglected. (Grønmo and Olsen, 2009).

2.4.1 "Mathematization" according to TIMSS

TIMSS gathered data from samples of the student population at three levels (middle primary, lower secondary, final year secondary), from the teachers of these students, and from their schools and systems.

Development of the tests of student learning outcomes for each student population began with an analysis of science curriculum guides and textbooks from many countries to 'identify priority topics' for the tests. An international panel of science curriculum specialists then produced a framework to guide test development.

The framework has the following dimensions:

- *a content dimension* which indicated the proportions of test questions required for each of the areas of science (life science, earth science, physical science, etc.), and
- *a performance expectations* dimension for what was likely to be involved in answering the items (understanding simple information, solving problems, using science processes, etc.).

There was also a *perspectives dimension* that included science-related details about the individual student and his/her classroom and school contexts (attitudes, interests, habits of mind, etc.).

Tests for different levels had different proportions of items on the content and performance expectations' dimensions, and included multiple choice, short answer and free response items.

The free response items usually required students to answer a question and then explain their answer. On a much smaller scale, some of the students from the first two populations undertook a 'Practical Performance Test' that required them to carry out a range of tasks and experiments involving simple scientific equipment. No comments are made about this performance testing in this digest as no international results have been published.

A School Questionnaire was used to gather the information about the intended and implemented curriculum, and a range of school characteristics (location, size, resources, curriculum offerings, etc.).
A Teacher Questionnaire asked about qualifications, levels taught, approaches to planning and teaching carrying out, use of textbooks and other resources, views on current curricular issues, etc.
Meanwhile, a Student Questionnaire sought information about demographic details, how students spent time, attitudes to science, expectations, etc. Finally, data for curriculum analyses were collected at the system-level, together with details on structural aspects that differed across the countries[1].

Performance expectations dimension

- Understanding simple information;
- Understanding complex information;
- Theorizing, analyzing, solving problems;
- Using tools, routine & science processes;
- Investigating the natural world.

2.4.2 "Mathematization" according to PISA

PISA puts the emphasis on mathematical literacy and 'preparation for life'. The beginning point for the testing is quite different to TIMSS. After consideration by the science expert group, the following definition of mathematical literacy was adopted by PISA for its testing:
"Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen."
This definition includes mathematical thinking and use of mathematical concepts, procedures, facts and tools for describing, explaining and statement of hypotheses about processes and phenomena. In other words, in mathematics PISA assesses students' skills to formulate, use and interpret mathematical problems in a variety of situations.
The following figure represents the mathematical literacy elements according PISA.

[1] TIMSS questionnaires have been published on the following address: http://nces.ed.gov/timss/questionnaire.asp

Mathematisation process

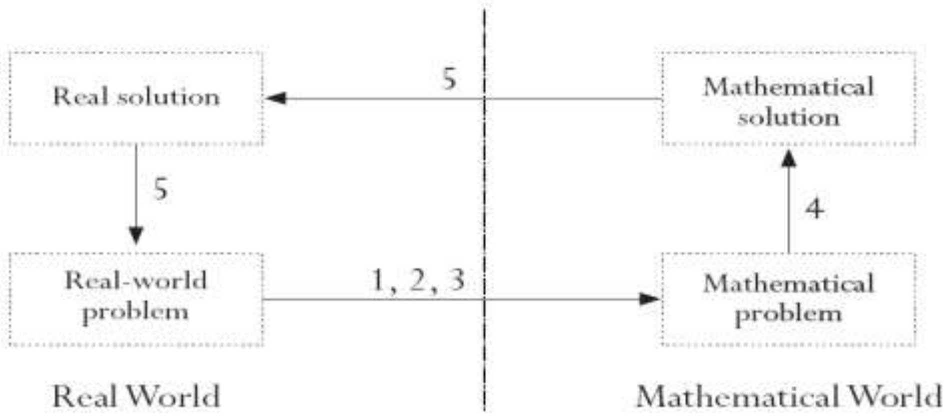

Figure 2: Mathematization process according PISA

Stage1

1. Starting with a problem situated in reality
2. Organizing it according to mathematical concepts
3. Taking into account the mathematical features of the situation and transforming real-life problem into mathematical problem

Stage 2

4. Solving the mathematical problem

Stage 3

5. Making sense of the mathematical solution in terms of the real situation, identifying the constraints of the situation.

Mathematization first involves translating the problem from "reality" into mathematics. This stage of the process includes activities such as:

- Identifying the relevant mathematics with respect to a problem situated in reality;
- Representing the problem in a different way; including its organization according to a mathematical concept with appropriate assumptions;
- Understanding the relationship between the language of the problem, symbolic and formal language needed for mathematical understanding;

- Finding regularities, relations and patterns;
- Recognizing aspects that are isomorphic with known problems;
- Translating the problem into mathematics; *i.e.* to a mathematical model, (de Lange, 1987, p. 43)

After translating the problem into a mathematical form, the next stage includes:

- Using and switching between different representations;
- Using symbolic, formal and technical language and operations;
- Refining and adjusting mathematical models;
- Combining and integrating models;
- Argumentation;
- Generalization.

The last step(s) in solving a problem involve reflecting of the whole mathematization process and the results.

- Understanding the extend and limits of mathematical concepts;
- Reflecting on mathematical arguments, and explaining and justifying results;
- Communicating the process and solution;
- Critiquing the model and its limits.

Mathematical literacy is assessed in the context of:

- *Mathematical content* which consists of four distinct content areas: 1) Quantities (including the use of numbers for quantitative characteristics and relations between real objects), 2) Space and forms (recognition of shapes in different configurations and sizes, looking for similarities and differences in the analysis of figures and their elements, knowledge of the properties of objects and their mutual disposition), 3) Functions and relations (mathematical description of different processes) and 4) Probabilities and data (working with data, graphical presentation and interpretation).
- *Mathematical competencies* which can be shown in operating successfully in mathematization within a variety of situations, extra- and intra-mathematical context, and can be identified with eight characteristics: 1) Thinking and reasoning, 2) Argumentation, 3) Communication, 4) Modelling, 5) Problem posing and solving, 6) Representation, 7) Using symbolic, formal and technical language and operations and 8) Use of aids and tools.

These characteristics define the main competency clusters of the mathematical competencies: *reproduction (*performing simple mathematical operations); *determine the logical relationships* (generalization of ideas to solve a problem); *reflection* (demonstration of mathematical thinking and reasoning).

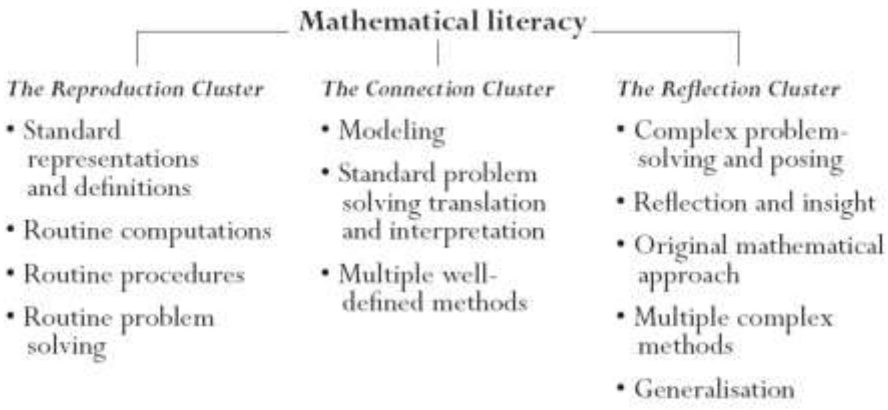

Figure 3: Competency clusters according PISA

The figure above represents the main competencies clusters and distinctions between them.

- **Measurement of cognitive processes:** formulate, use, interpret.

2.4.3 "Mathematization" concept and Arts integration

The use of an adequate language for the formalization of mathematical concepts is crucial in the school education. According to PISA mathematization follows a dynamics that goes from the real world problem towards it's translation into the mathematical language and from the mathematics solution to the real world solution.

When implementing learning in applied mathematics, the real world problems are mathematized into the abstract form which represents them. Also in implementing STEAM learning, the situation is the same with learning content being "mathematized" or transferred into the abstract form of any art representing it.

The art becomes a unifying element among different languages, such as visual, sensory, verbal and non-verbal which can encourage the development of both cognitive and emotional dimensions. For this reason it becomes an important element for the harmonious development and growing up of human being (Dallari 1998; Montessori 1992).

Moreover, the arts support students in better connecting of scientific subjects and reality by rediscovering their usefulness and their application in everyday life. Therefore, the learners become active in their learning process by constructing of own knowledge in "meaningful" context through the objects manipulation, tools and through the observation and interpretation of their actions' results. In these terms the meaningful learning gets contextualized and complex. The students learn more and better if they cope with authentic tasks directly connected to the real world, where they met their everyday lives' "real problem". The same thing occurs when student approaches to STEM subjects. Actually, people often forget that reality as well as all disciplines known by man is based on scientific concepts, where mathematics holds a prominent position.

For example, with the introduction of the Fibonacci series (Posamentier et al. 2010), made by the Medieval great mathematician Leonardo da Pisa, it is possible to observe how the growth of flowers, leaves and branches of a plant follows the specific patterns. In crystals (Gallo 2007), the symmetry elements regulate the arrangement of the various faces and edges in crystal lattices, and this has enabled the classification of the crystals, for example the cubic form in the minerals such as halite or trigonal form in the dolomite. Despite of this strong presence of mathematical concepts in the reality, their connection doesn't appear so evident during the learning process and often teachers offer to students a one-sided theoretical approach causing the perception that mathematics is abstract and far away from everyday life. This may inevitably influence a way the mathematics is learnt, namely privileging more storage capacity than problem solving skills.

3. Analysis of national school curriculum and art-works selection

The procedure of the identification of the G.A. STEM exercises has started by an analysis of national educational standards in mathematics and science described in national curricula of all the countries involved in the educational research process of the G.A. STEM Project – Finland /FI/, Belgium /BE/, Italy /IT/, and Estonia/EE/. This chapter represents the main results of the national curricula of Math/Science for 13-16 years old students.

The results of the curricula analysis help the G.A.STEM RET to determine what are the common aspects and objectives of the training in Math and Science on international level. The identified common objectives could be summarized as follows:

1. Recognizing and understanding the terms of geometry.
2. Deepen and broaden students' knowledge of geometry figures by identifying invariants and relationships and studying the mutual positions of the circle and cylinder,angle, and polygon, and properties of the remarkable points in a triangle.
3. Using the Cartesian plan to represent relationships and functions empirical or derived from tables, or their graphs, and then link them to the concept of proportionality.
4. Deepening of knowledge and logic skills, formation of logical culture and learning of mathematical language.
5. Forming a positive attitude towards mathematics, creating interest and motivate students to acquire knowledge and skills.
6. Development of observation, imagination, concentration of the thinking, memory.
7. Deepening of knowledge in reflection and absorption of light and their real applications.
8. Knowing connections between adaption and physical-chemical conditions/environment.
9. Identifying the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships;
10. Let students recognise the link between mathematics and art through the symmetry study.

In order to select assessment schedules on international level G.A. STEM RET needed common framework defining the indicators and attributes for the description of the scientific topics covered by national educational curricula. The selected set of attributes allows RET to analyze specifics of the national standards but also to render an account the compliance with the indicators of TIMSS and PISA programs.

Concerning the exercises representation by RET were selected two main classes of attributes:

- Expected results – this class of attributes covers the following:
    - Description of nucleus of the learning content (macro area)

  - Description of the expected results at a curriculum level
- Learning content – this class of attributes aims to define the standards that students must meet as a result of completion of the appropriate level of school curriculum; the outcomes that students must achieve and the opportunities that provide the curriculum. In detail, it describes the expected results divided into topics, the basic terms divided into topics and the context and activities to be learnt by students.
- Art-work selected – this class describes the art-works corresponding to the mathematics and science topics selected for the G.A. STEM students.

On the basis of selected attributes RET developed common template for description of the topics covered by national curriculums of the partner countries.

The template structure is presented on the figure below.

Expected results		Learning content (topics, concepts, context and activities)			Art-Work selected
Nucleus of the learning content	Expected results at a curriculum level	Expected results divided into topics	Basic terms (divided into topics)	Context and activities	

Figure 4: Attributes for Math and Science topics (exercises) description

The descriptions provided by partners are presented in APPENDIX 1 of this document.

3.1 Selection Procedure

Specification of common criteria for selection of international and national schedules is based on the results of both the international assessment instruments and schemes more concretely TIMSS and PISA and the national educational curricula of the countries involved. The selection of assessment schedules was based of unified descriptions of national educational curricula. Provision of such unified set of attributes allows RET to select appropriate and relevant assessment schedules which are in accordance of the national standards.

The procedure for selection of international and national assessment schedules had several stages:

Stage1: Development of the template for unified description of the mathematics and science topics covered by educational curriculums of the partner countries.

Stage2: Each partner country provides 2 to 4 exercises which are:

- Relevant to the described Mathematics/Science topics in terms of:
    - Standards that students must meet as a result of completion of the appropriate level of secondary school;
    - Outcomes that students must achieve;
    - Opportunities that provide the curriculum.
- Adaptable for 13-16 years old students;
- Good combination between mathematics/science and art works.
- Accordance with PISA Competency Clusters and their concrete semantics in terms of thinking and reasoning; argumentation; communication; modeling; problem posing and solving; representation; using symbolic, formal and technical language and operations; use of aids and tools.

Stage3: RET of each partner country revises the proposed exercises in respect to the criteria described above.

Stage4: RET of each partner country votes each of the proposed exercises in order to select these which are appropriate to be realized into the G.A. STEM products. There were two rounds for voting in order to reach the total number required, that is n. 10 exercises.

Stage5: The selected exercises where those received a maximum number of votes.
The results from stages 3 and 4 of the selection process are briefly presented in the table below:

	Proposed exercises
FINLAND	1. Finland - Fibonacci
	2. Finland - Harmonic Series
	3. Finland - Snow crystal
ITALY	1.Italy - Naumachie
	2.Italy - Obelisco
	3.Italy - Un giudizio universale
ESTONIA	1. Estonia - Plan mirror
	2. Estonia - Mondrian art
	3. Estonia - Ant-Man and Science
	4. Estonia - Estonian Ornament
BELGIUM	1. Belgium - Fibonacci
	2. Belgium – Pythagoras
	3. Belgium - Symmetry

Table 1. Results of proposed exercises.

Next table presents the final results from the selection process.

FINLAND	**International**
	1. Estonia - Plan mirror
	2. Estonia - Mondrian art
	3. Estonia - Ant-Man and Science
	4. Estonia - Estonian Ornament
	5. Belgium – Pythagoras
	6. Belgium - Symmetry
	7.Italy - Naumachie
	8.Italy - Un giudizio universale
	National
	1. Finland - Harmonic Series
	2. Finland - Snow crystal
ITALY	**International**
	1. Estonia - Plan mirror
	2. Estonia - Mondrian art
	3. Estonia - Ant-Man and Science
	5. Belgium – Pythagoras
	6. Belgium - Symmetry
	7. Finland - Harmonic Series
	8. Finland - Snow crystal
	National
	1.Italy - Obelisco
	2.Italy - Un giudizio universale

ESTONIA	**International**
	1. Belgium – Pythagoras
	2. Belgium - Symmetry
	3.Italy - Naumachie
	4.Italy - Un giudizio universale
	5. Finland - Harmonic Series
	6. Finland - Snow crystal
	National
	1. Estonia - Plan mirror
	2. Estonia - Mondrian art
	3. Estonia - Ant-Man and Science
	4. Estonia - Estonian Ornament
BELGIUM	**International**
	1. Estonia - Plan mirror
	2. Estonia - Mondrian art
	3. Estonia - Ant-Man and Science
	4. Estonia - Estonian Ornament
	5.Italy - Naumachie
	6.Italy - Un giudizio universale
	7. Finland - Harmonic Series
	8. Finland - Snow crystal
	National
	1. Belgium – Pythagoras
	2. Belgium - Symmetry

Table 2-3 Summarized final results from RET voting

4. Analysing the existing practices of the use of the games for math and science study for 13-16 years old students

4.1 Use of the games in learning

Based on their study's literature review Perrotta, Featherstone, Aston and Houghton (2013) describe game-based learning as an experiential engagement of learning by trial and error, role-playing and treating a certain topic not as 'content' but as a set of rules or choices and consequences. In a school curriculum, this means translating an element of a subject (such as a law of physics) into the mechanics of a game which functions independently in its own system, based on choices and consequences. (Perrotta et al. 2013) Citing Trybus (2015) Pho and Dinscore (2015) clarify the same idea by defining that concept of game-based learning refers to the borrowing of certain gaming principles and applying them to real-life settings for users' engagement.

According to Rivera (2016) most people need effective and interactive experiences that motivate and activate them during the learning process, and game-based learning offers one means for that. Game-based learning today involves the use of computer and video games specifically aimed for the learning purposes.

4.2 Learning and educational outcomes

According to Sherry (2013) around 2000 there started to rise a new interest in using computer games for learning in various subjects, particularly in STEM subjects. The games brought into learning the new opportunities to personalize learning, to simulate different phenomena and to do many things that otherwise weren't possible. At the same time there was also a lot of empirical research linking digital game play together with effective STEM content learning and problem-solving. However the real evidence for how STEM game play transfers to durable learning was still lacking, and existing research had contributed little to a more general understanding of how children learn from games. The insufficiency of the studies and their results concerning games and learning has been noted also in academic journals and policies. Among the key research questions that remain unanswered are: (1) the specification of the game aspects, game mechanics, and formal features that are mostly connected to learning content within digital games, (2) examination of how the former might differ by age, (3) how learning content is interpreted and understood in the game context, (4) the role of others (such as teachers) in learning from games, and (5) the possibilities of transferring content from games to nongame contexts. (Sherry 2013).

According to Hamari, Shernoff, Rowe, Coller, Asbell-Clarke and Edwards' (2016) study on game-based learning, educational video games may be effective in posing learning challenges that are perceived as interesting, enjoyable, and awaking students' engagement and immersion in the game-based learning task. Especially the challenge created by the game appeared to be an important factor in students' engagement, and thus essential for learning through the game. The findings of the study suggest that game designers should focus on challenge and engagement, and at the same time consider players' skills which were also found to play an important role in engagement and immersion. According to the study, the ability of high quality educational video games to offer complex problems to be solved both challenging and enjoyable way is a strong base on which to build when developing learning environments. (Hamari et al. 2016).

In their study Bourgonjon, Valcke, Soetaert and Schellens (2010) wanted to examine the debate whether today's students, also referred to as millennials, digital natives and game generation, need a technology based and whole different kind of learning approach than traditional classroom learning or not. They proposed and empirically tested a path model to examine and predict student acceptance of video games with 858 secondary school students. The results show that students' preference for using video games in the classroom is affected directly by a number of factors: the perceptions of students regarding the usefulness, ease of use, learning opportunities, and personal experience with video games in general. Gender effects are found as well, but appear to be mediated by experience and ease of use. (Bourgonjon et al. 2010).

4.3 *Analysing the existing practices of the use of the games for math and science study*

In their research Beier, Miller and Wango (2012) state that both the exposure to science concepts as part of using game applications with virtual apprenticeship component, and the influence of a peer group who share similar interests, may support students' motivation and interest in STEM careers. Through an example of a forensic game CSI: THE EXPERIENCE, as part of 8th grade biology unit studying DNA, they described the importance of exposing students to career opportunities associated with STEM subjects. The research suggests that further enthusiasm over STEM relies heavily on initiating interest and fostering engagement in science by middle school. (Beier et al. 2012).

According to Cheugn and Slavin's (2013) study of the effectiveness of educational technology applications for enhancing mathematics achievement in K-12 classrooms, the educational technology's positive effect to learning mathematics, when compared to traditional methods, is noticeable, but modest. Based on their review in which total of 74 qualified studies were included in their final analysis with a total sample size of 56,886 K-12 students (45 elementary studies; N = 31,555 and 29 secondary studies; N = 25,331) they suggest that new and better tools are needed to reach a goal of enhancing children's mathematics achievement with technology. The review indicates that incorporating supplemental programs into regular classroom curriculum may be beneficial. (Cheugn & Slavin 2013).

According to Denner, Werner and Ortiz (2012) more attention has been put into creating game programming environments than into understanding children's learning process and its connections to these environments. The findings of their research analysis of 108 games created by middle school girls in an after school class show that students engaged in moderate levels of complex programming activity, created games with moderate levels of usability and low levels of code organization and documentation. Based on the results it seems that game construction involving both design and programming activities can support the learning of computer science concepts, offering students the opportunity to engage in the kind of thinking that will prepare them for further study in computing. However, among students with no prior experience in programming, more extensive instructional support is needed to achieve the learning results with the more complex computer science concepts, creating and understanding organizational systems, as well as to identify and remove faults in their programs. (Denner et al. 2012).

In their wide literature review on personalized learning in middle school mathematics education Evans, Pruett, Chang and Nino (2014) identified three priority areas that should be taken into account when successfully adopting learning games for the classroom. Those areas were personalized feedback, learning assessment and deepening of students' learning. This is why it is important for teachers to be involved in game design, for instance, in the form of co-design, to be interpreting the designers their teaching and instructing practices. Also collecting data on using learning games in the classroom as case evidence may provide important aspects and help for future developments in the field. (Evans et al. 2014).

Also according to Kwon, Lara, Enfield and Frick (2013) guidance in the learning process is important. Especially for non-experienced learners instructions help their understanding the game objectives, rules and interfaces and protect the learners from suffering cognitive overload. Also clarification about the context of the game and any difficult terms used within it help especially the students with weaker language skills. Educational games, especially complex games, are usually more text-based than graphics-based and language barriers or learners' cultural backgrounds may prevent the learners enjoying the game or meeting the goal of the game. (Kwon et al. 2013).

Ke's (2013) mixed-method case study examined the potential of computer-assisted, math game making activities in facilitating design-based math learning for school children with a test group of 64 middle school children participated in Scratch-based, math game making activities. According to the study findings, the game design promoted the participants' more positive attitude toward mathematics significantly and engaged them in thinking mathematically, even though the math content was less at the centre of students' attention than the storytelling, game-world crafting and programming. (Ke 2013).

In Kebritchi, Hirumi and Bai's (2010) study of the effects of a computer game on students' mathematics achievement and motivation, and the role of prior mathematics knowledge, computer skill, and English language skill on their achievement and motivation as they played the game, it was found that games had a significant positive effect on students' mathematics achievement in the public high school setting. The students who played the games also scored significantly higher on the district-wide math benchmark exam than the ones that did not play them. Teachers and students' interviews support the quantitative findings, as most of the both groups of participants reported on improved mathematics understandings and skills as a result of playing the mathematics games. According to the teachers, the games were effective teaching and learning tools because they offered experiences, alternative ways of teaching, reasons for problem solving, diminished students' mathematics phobia, and increased time on task. According to the students, the games were effective because they made learning fun, brought possibilities for adventure and exploration, and gave challenges into learning mathematics. Also collaboration with other students was mentioned to be more attractive than playing alone. These findings are in line with former empirical research on the effects of math games. Additionally they support the findings of studies that suggest that interactive games are more effective on learners' cognitive gains than traditional instruction, as well as the learning effectiveness of the experiential learning theory developed by Dewey (1938), and elaborated by Kolb (1984). Although the mathematics games don't include authentic problems, issues and experiences, they provide hands-on activities and simulated missions that engaged students in learning by doing and experiencing. However, because of the other factors included in the game situation and game environment, further investigation may better test the effectiveness of the experiential theory. (Kebritchi et al. 2010).

In their review of the literature aimed at examining the use of video games as part of K–12 curricula Young, Slota, Cutter, Jalette, Mullin, Lai, Simeoni, Tran, and Yukhymenko (2012) state that video gaming in mathematics is characterized by specialized math games developed within and for controlled studies, cited in only a small number of articles and not used in further research or replication studies. In the review they state that even though the findings of the studies report positive outcomes such as increased students' achievement and motivation, these results are not unambiguous. According to the review "educational gaming in mathematics is not as simple as putting a student in front of a computer" but rather "educational games need to be designed and researched with careful attention to contemporary learning theories, including customization of task difficulty to the learner's capabilities, metacognitive reflection on the learning taking place, and consideration of the rich situated interaction among learner, game environment and classroom environment" and that is why the longer-term future research for field's further examination is recommended. (Young et al. 2012).

Examining game-based learning for science, Young at al. (2012) note that despite a decade of research emphasis on STEM education, there has been little peer-reviewed literature published, and the existing literature, such as that concerning mathematics, is not consistent when it comes to monitoring activities, assessing learning outcomes in terms of monitoring activities, assessing learning outcomes, or noticing other variables of science-based gaming. From 11 science video gaming studies relevant to the review's objective only 5 contained any empirical data with academic achievement listed as the primary dependent variable. The most positive outcomes came from a study by Barab, Goldstone, and Zuiker (2009) and two related dissertations by Zuiker (2008) and Arici (2009) concerning the science-based game Quest Atlantis. As the achievement of the game-playing middle school students (Grades 6–8) was significantly better on related standardized tests than the non-playing peers, the authors became interested in furthering game-based research in STEM fields. However, according to the review and the studies examined, citing Baek, Kim, & Park (2009), Young et al. (2012) mention it seems a good idea to implement science games only combined with good teaching, so the teachers can help transferring of science information from isolated in-game activities by encouraging student reflection, providing questions to reference the specific curricular science skills, and help content's connections with real life. Finally, based to the review, linking science achievement conclusively to game use needs more research in the field of science-based gaming, including more sophisticated, empirical studies to capitalize on the learners' interactions, virtual and real environments, game mechanics and narrative elements, and the integrated nature of the sciences. (Young et al. 2012).

In another wide review is the one of Connolly, Boyle, MacArthur, Hainey and Boyle (2012) in which they examined the literature on computer games and serious games in regard to the potential positive impacts of gaming on users aged 14 years or above, participally conserning learning, skill enhancement and engagement. Search terms identified 129 papers reporting empirical evidence, based on which a multidimensional approach to categorizing games was developed. According to the findings game playing is linked to a range of perceptual, cognitive, behavioural, affective and motivational impacts and outcomes, most frequent being knowledge acquisition/content understanding and affective and motivational outcomes. To encourage the use of games in learning beyond simulations and puzzles, a better understanding of the tasks, activities, skills and operations that different games can offer and examine these in relation to the desired learning outcomes. Also the detail of how games are integrated into the student's learning experience is an essential question. (Connolly et al. 2012).

Because of the continuing interest in digital games Boyle, Hainey,Connolly, Gray, Earp, Ott, Theodore Lim, Ninaus, Ribeiro and Pereira (2016) updated Connolly etal.'s (2012) systematic literature review of empirical evidence about the positive impacts and outcomes of games. The updated review focused on 143 newer papers that provided higher quality evidence about the positive outcomes of games. According to the review, the term "serious games" has been used as a synonym for learning games, games for learning have been used to promote knowledge acquisition across a wide range of topics, and it has been accepted that it can be more useful to develop games that address specific curricular objectives. Despite the intense interest in games, it is important to keep in mind that the field is complex and costly and still has a lot of challenges. Based on the review, future research on digital games would benefit from a systematic and detailed research on which game features are most effective in promoting engagement and supporting learning. (Boyle et al. 2016).

5. Analysing game design and development elements

5.1 Analysing game design

Citing different studies, Álvarez-Rodríguez, Barajas-Saavedra and Muñoz-Arteaga (2014) describe three different models of a game design process, first of which consists of the following steps: 1) Developing the core idea; 2) Writing a game concept; 3) Producing the artwork; 4) Programming the game engine; 5) Game content production; 6) Play testing; and 7) Balancing and bug fixing (Masuch & Rueger 2005). The second model, educational game design model, divides the game production into the main stages of a) game design, b) pedagogy, and c) learning content modeling (Ibrahim & Jaafar 2009. The third educational game design process consists of four main elements, interaction, knowledge, engine and level (Zin & Yue 2009). However, none of these described models have a structured process that guides to the reader from a starting point in the process to the end where a game is a finished product. (Álvarez-Rodríguez et al. 2014).

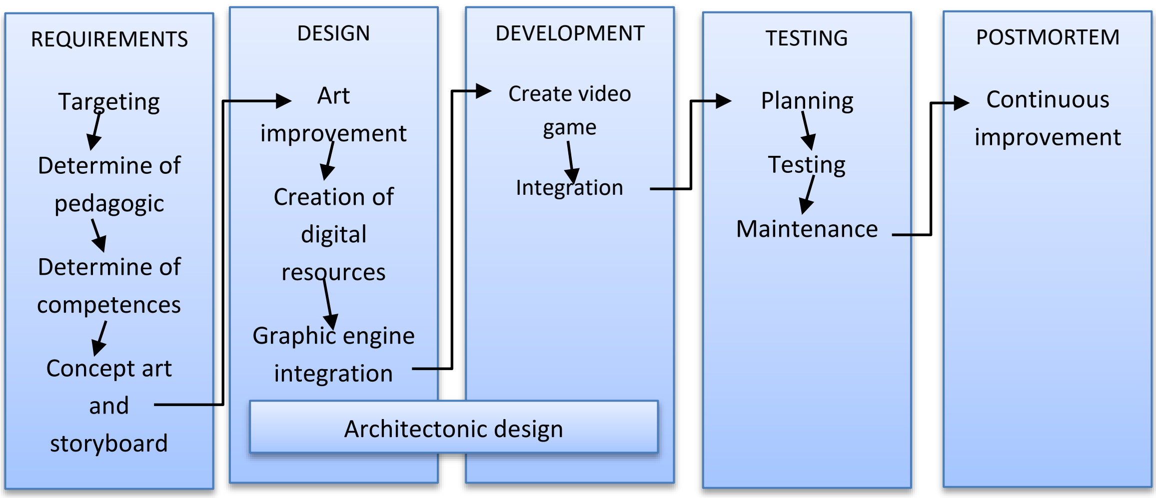

Figure 5: Game Design Process proposed. Copied from Álvarez-Rodríguez et al. 2014.

In Figure 5 there is thes by Álvarez-Rodríguez et al.'s (2014) proposed Game Design Process with five stages, which are intended to capture all the elements of the game: the targeting and pedagogical principles, the design and development, technical and usability testing, testing for absorption of knowledge, and finally continuous improvement. This Game Design Process is based on the traditional Software Engineering paradigm, the large scale development of digital learning resources (Barajas Saavedra, 2009), and specific skills on Mathematics. The process moves through stages as described below:

- At Requirements Stage the goals of the game are to be set, the pedagogic mechanisms, across which the knowledge will be transferred to the students are to be established; the competences and the knowledge areas that must be covered are to be determined; and storyboard and concept art are to be created.

- At Design Stage all digital resources needed by the game engine for the creation of the video game are to be created, including 2D illustrations, 3D models, Maps, Objects, Materials, surfaces, etc., Sounds and music; and to create game engine if needed.

- At development Stage the game, including Layout, Events, Shader, and AI is to be created, the game play is to be designed, and all the above elements are to be integrated with menus, options, etc.

- At testing Stage the videogame is to be tested by its Technical, Knowledge absorption, Usability, Usefulness aspects; efficiency statistics is to be obtained; and game is to be to maintained.

- At Postmortem Stage all process and product information collected during development process are to be collected to improve future developments.

According to Álvarez-Rodríguez et al. (2014) advantages of the Proposed Process are that 1) The game development process proposed provides developers and game designers a process which leads the developers clearly through the production of a serious video game, with a map of the steps from conception of an idea to the release of the game, 2) The game development process provides a framework for the integration of multidisciplinary experts to develop a serious video game together, 3) The game development process proposed is developed in a transparent way because the game is considered as a software product, so that a company dedicated to software development can deploy it easily and efficiently, and it is implemented to be independent of the product type to be developed, and 4) the game development process also provides the ability to integrate the pedagogics and competencies to the video game requirement efficiently, so the video games designed can meet the need in the classroom and the teachers will get guidance on how to integrate the game into their teaching in formal education. (Álvarez-Rodríguez et al. 2014).

5.2 Analysing game development elements

Sillaots, Jesmin and Rinde (2015) point out that games can be structured by their game elements which are like building blocks that create a game experience. Game elements can be combined into different sets, and some of the design elements are sub-elements for designing higher level game aspects. By using game elements it is possible to create game patterns like intrinsically motivating experiences. (Sillaots et al. 2015).

According to Sillaots et al. (2015) there are different views about what elements the games can have and how to categorize them. In game design and gamification the conceptual map of game elements can be used for example in classroom conditions when willing to make the game or learning activity as engaging (or enjoyable, involving, immersive, entertaining, fun, motivating) as possible. In this case the reading of the concept map of game elements (see Figure 2 on the next page) should start from the top. At first, game designers should determinate the desired level of engagement. After the level of engagement is set, the game elements that ensure the desired quality are to be selected. It is not necessary to design environments that generate the actual state of flow; at start it is enough to select flow-enabling factors (balance between competences and challenges, clear goals, feedback, concentration and control). After this it can be decided the competences, the challenges, the feedback and the rules that the game or gamified environment should provide. And finally it is time to consider what this environment should look like to bring alive all the ideas that were created. For example, where does the interaction take place: between players (socialization) or between game environment (world and NPC's) and story? (Sillaots et al. 2015).

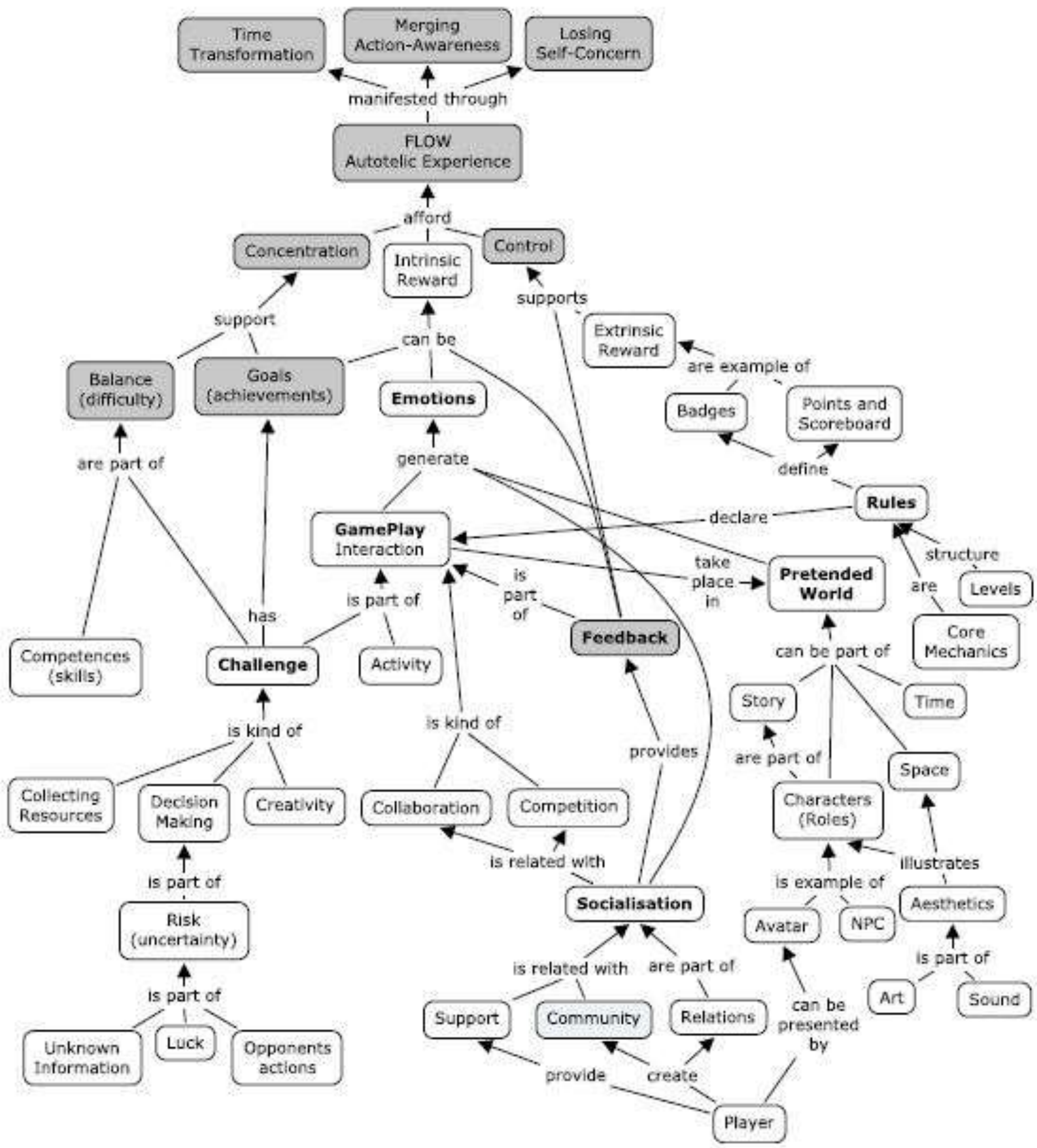

Figure 6: Concept map of game elements. Copied from Sillaots et al. 2015.

APPENDIX 1 – Unified Math and Science topics description

Math and Science topics selected by Finnish RET

Expected results		Learning content (topics, concepts, context and activities)			Art-Work selected
Nucleus of the learning content	Expected results at a curriculum level	Expected results divided into topics	Basic terms (divided into topics)	Context and activities	
Plane geometry	• Recognizing and understanding the terms of geometry; • Being familiar with concept of symmetry.	• Knowing and mastering the concept of angle; • Able to use the properties of triangle with other polygons; • Being familiar with concept of polygon and regular polygon; • Able to recognize symmetry in polygons; • Able to solve problems concerning angles in regular polygon using triangles.	• Angle, degree, full angle, triangle; • polygon, regular polygon, hexagon; • symmetry, central symmetry	• The student knows how to divide hexagon in similar triangles; • The student calculates the angles of a polygon using the knowledge of sum of triangles angles. • The student finds the symmetry point in regular polygons and uses symmetry when solving angles.	Snow crystal

Expected results		Learning content (topics, concepts, context and activities)			Art-Work selected
Nucleus of the learning content	Expected results at a curriculum level	Expected results divided into topics	Basic terms (divided into topics)	Context and activities	
Frequency	• Pupils understand the formation of human voice; • Pupils understand the formation of the sounds of different instruments.	• Pupils understand that the human voice contains always both the fundamental frequency and overtones. • Pupils understand that the sound of instrument contains always both the fundamental frequency and overtones	• Frequency, fundamental frequency, overtone, tone color • Frequency, fundamental frequency, overtone, tone color	• Recognise human voices only by the tone color • Recognise instruments only by the tone color	Christmas Carol: "Angels We Have Heard on High"
Fractions					
Series					

Math and Science topics selected by Italian RET

Expected results		Learning content (topics, concepts, context and activities)			Art-Work selected
Nucleus of the learning content	Expected results at a curriculum level	Expected results divided into topics	Basic terms (divided into topics)	Context and activities	
Cartesian coordinates	• Using the Cartesian plan to represent relationships and functions empirical or derived from tables, or their graphs, and then link them to the concept of proportionality.	**Topic: building relationships.** The student in various contexts, identifies, describes and builds significant relationships: recognizes similarities and differences. **Topic: Cartesian Plan** The student uses the Cartesian Plan to represent relationships and links them to the concept of proportionality.	• Using mathematic formulas applied to the geometry. • Making calculations using calculation tools. • Appling problem solving techniques. • Using the theory of numerical approximation.	The student is able to: 1) critically observe objects of daily life; 2) represent geometric shapes; 3) identify and interpret data; 3) carry out calculations to obtain numerical results, approximated if necessary, with appropriate units of measurement; 4) apply theorems and use the Cartesian plan; 5) recognize and manage the	Michelangelo's "Giudizio Universale"
Pythagoras Theorem	• Calculating the sides of a rectangle triangle using the Pythagorean	**Topic: Use of formulas and calculations.** The student calculates the length of the	• To adequately justify statements, distinguishing between statements induced by observation, intuited		

	Theorem also starting from real situations of everyday life.	hypotenuse using formulas and theorems. **Topic: Pythagoras' theorem.** The student knows the Pythagorean theorem and its applications in mathematics and in concrete situations	and hypothesized, argued and demonstrated.	proportional relationships between different representations; 6) transfer mathematical calculations to real cases obtaining concrete answers.	
Ratios and proportions		**Topic: Visualization and representation.** The student visualizes and represents two-dimensional geometric figures from a real artistic representation. **Topic: Proportionality.** The student, starting from a representation of reality, grasps the proportionality between the real and the represented and manages the measures			Michelangelo's "Giudizio Universale"

		and its units.			
Expected results		Learning content (topics, concepts, context and activities)			Art-Work selected
Nucleus of the learning content	Expected results at a curriculum level	Expected results divided into topics	Basic terms (divided into topics)	Context and activities	
Solid geometry: the cylinder	• Comparing and analysing geometric figures identifying invariants and relationships; • Calculating the area and volume of circle and cylinder; • Giving estimates of objects of everyday life.	**Topic: visualization and representation.** The student visualizes three-dimensional objects starting from a two-dimensional representation and vice versa, to represent a solid figure on a plane. **Topic: use of formulas and calculations.** The student: calculates lengths of circles, areas of circles and the volume of the cylinder. **Topic interpretation.** The student interprets constructs and transforms formulas that contain letters to express relations and	• Using mathematic formulas applied to the geometry; • Making calculations by using calculation tools; • Applying problem solving techniques; • Using the theory of numerical approximation.	The student is able to: 1) represent geometrical shapes; 2) identify and interpret data; 3) carry out calculations to obtain numerical results with units of measurement; 4) transfer mathematical calculations to real-life cases obtaining concrete answers.	Monumental complex called Septa in which Augustus (Roman emperor) organized the most famous Naumachia.

		properties in general form. **Topic: use of the fundamentals of Euclidean geometry.** The student knows and uses the basics of the plane and the geometric space.			

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Math and Science topics selected by Estonian RET

Expected results		Learning content (topics, concepts, context and activities)			Art-Work selected
Nucleus of the learning content	Expected results at a curriculum level	Expected results divided into topics	Basic terms (divided into topics)	Context and activities	
Reflexion of light	• Knowing the important characteristics of the reflection and absorption of light and uses them in practice; • Naming the significant characteristics of the concepts like *angle of incidence, angle of reflection;* • Explaining the law of reflection (i.e. in case of the reflection of light the angle of reflection is equal to the angle of incidence) and its meaning; • Describing the	• Knowing the important characteristics of the reflection and absorption of light, describes the connection with other phenomena and uses them in practice; • Naming the significant characteristics of the concepts like *angle of incidence, angle of reflection;* • Explaining the law of reflection (i.e. in case of the reflection of light the angle of reflection is equal to the angle of incidence) and its meaning; • Describing the experiment proving its validity and uses the relation in practice;	• Law of reflection, • Angle of incidence, • Angle of reflection, • Incident ray, • Reflected ray, • Plane mirror, object, image	1. Drawing and improving the drawing according to instructions; 2. Carrying out an experiment; 3. Creating	Painting: Arnolfini' spouses (1434) by Jan Van Eyck

	experiment proving its validity and uses the relation in practice; • Giving examples of the usage of the plane mirror.	• Giving examples of the usage of the plane mirror.			
Expected results		Learning content (topics, concepts, context and activities)			Art-Work selected
Nucleus of the learning content	Expected results at a curriculum level	Expected results divided into topics	Basic terms (divided into topics)	Context and activities	
Physics: Mechanics - motion and force	• Knowing connections between adaption and physical-chemical conditions/environment.	• Explaining the meaning of length, area, and density and the ways to measure them; • Knowing the units of measurement; • Knowing the meaning of relation and uses it while solving problems; • Carrying out an experiment measuring the test body mass and volume; • Processing test data;	**Physics: Mechanics** • Motion and force (mass, density); • Units of measure: length, volume, area, density; • Test data; **Science**: • Density, readjustment and adaptation; • Diversity of matter; • Relations between the living and non-living	1. Recalling necessary relations; 2. Making sketches (draws); 3. Looking for and upgrades information; 4. Analysing, discussing, drawing conclusions; 5. Modelling	Action Figures M. C. Escher's paitings: - Perpetual Motion Waterfall - Ascending and Descending - Relativity

Co-funded by the Erasmus+ Programme of the European Union

		• Making necessary calculations based on the test results; • Comparing the data in table.	nature.	with plasticine.	
Expected results		Learning content (topics, concepts, context and activities)			Art-Work selected
Nucleus of the learning content	Expected results at a curriculum level	Expected results divided into topics	Basic terms (divided into topics)	Context and activities	
Percentage. The Golden Ratio. Area.	• Expressing the division of two numbers or ratio in percentage; • Understanding the concept of a ratio and uses ratio language to describe a ratio relationship between two quantities; • Calculating what percentage one number is of another and explains what the result shows;	**Percentage**: • Expressing the division of two numbers to a ratio in percentage; • Calculating what percentage one number is of another and explains what the result shows. **Ratio:** • Recognizing and representing proportional relationships between quantities; • Identifying the constant of proportionality (unit	Expressing ratio as a percentage. The Golden Ratio. Area	1. Solving the tasks, to calculate the perimeter and area of the given geometric shapes; 2. Solving the tasks to find the ratios and percentages; 3. Creating own Modrian Art.	Mondrian Art

	• Recognizing and representing proportional relationships between quantities; • Identifying the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships; • Solving real-life and mathematical problems involving area.	rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships; • Understanding the concept of a ratio and uses ratio language to describe a ratio relationship between two quantities. **Geometry** • Solving real-life and mathematical problems involving area.			
Expected results		Learning content (topics, concepts, context and activities)			Art-Work selected
Nucleus of the learning content	Expected results at a curriculum level	Expected results divided into topics	Basic terms (divided into topics)	Context and activities	
A polygon. Quadrilaterals.	• Knowing what is a polygon; • Noticing mathematical shapes in	**Geometry - Polygons** • Knowing what is a polygon;	• A polygon. • A parallelogram. • A rhombus.	1. Recognising geometric shapes; 2. Drawing an ornament on	Estonian Ornament

	everyday life; • Understanding that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; • Verifying experimentally the properties of rotations, reflections, and translations.	• Noticing mathematical shapes in everyday life. **Geometry – Transformations** • Understanding that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; • Verifying experimentally the properties of reflections and translations.	• Reflection.	paper; 3. Drawing an ornament using Geogebra instructions; 4. Designing an ornament with Geogebra	

Math and Science topics selected by Belgium RET

Expected results		Learning content (topics, concepts, context and activities)			Art-Work selected
Nucleus of the learning content	Expected results at a curriculum level	Expected results divided into topics	Basic terms (divided into topics)	Context and activities	
Symmetry Coordinates Appreciating Flemish art	**Standard 1:** Pupils can solve problems using the right heuristics. **Result:** Pupils can detect a pattern or symmetry in a situation. **Standard 2:** Pupils can determine symmetry lines in geometrical shapes. **Result:** Pupils can draw the symmetry line. **Standard 3:** Pupils recognize the link between mathematics and art. Result: They can draw a	**Topic 1:** Pupils detect the symmetry in given figures. **Topic 2:** Drawing symmetry lines in regular polygons. **Topic 3:** Pupils are introduced to the Ghent Altarpiece and learn about its history. Pupils draw symmetry lines in pictures of architectural buildings in Ghent. **Topic 4:** Pupils pinpoint locations using coordinates on the map of Ghent.	• Pupils recognize symmetry. • Pupils draw symmetry lines in regular polygons. • Pupils recognize the link between art, architecture and mathematics. • Pupils find coordinates on a given map.	• Pupils take a virtual tour through Ghent. • Pupils recognize symmetry lines in iconic buildings of Ghent. • Pupils recognize different polygons. • Pupils appreciate Flemish art. • Pupils use given coordinates to find locations on a map. • Pupils are able to draw regular polygons. • Pupils draw the symmetry lines in the regular polygon. • Pupils find the	Ghent Altarpiece

	symmetry line on a picture of an architectural building. **Standard 4:** Positioning points in the plain using coordinates. **Result:** They use coordinates to find a location on a map.			intersection of the symmetry lines. • Pupils can determine the coordinate of the intersection of the symmetry lines of the regular polygon.	
Expected results		Learning content (topics, concepts, context and activities)			Art-Work selected
Nucleus of the learning content	Expected results at a curriculum level	Expected results divided into topics	Basic terms (divided into topics)	Context and activities	
Pythagorean theorem	**Standard 1:** Pupils can formulate the Pythagorean equation. **Results:** Pupils recognize Pythagorean theorem in right triangles and use it in exercises. **Standard 2:** Pupils discover different ways to proof the Pythagorean theorem.	**Topic 1** • Pupils know that the Pythagorean Theorem can only be used in a right triangle. • Pupils can formulate the Pythagorean equation with any given right triangle. **Topic 2** • Pupils recognize that a problem can often be	• Pupils can recognize right triangles when the sides are given. • Pupils use ICT accordingly.	• Pupils know the theory behind the Pythagorean Theorem. • Pupils draw a Pythagorean tree using ICT. • Game element: pupils	Pythagorean Tree

	Result: Pupils use a large range of applets and applications to research the Pythagorean theorem. **Standard 3:** Pupils use the Pythagorean theorem in geometrical constructions. **Results:** Pupils draw the Pythagorean Tree. **Standard 4:** Pupils use basic and a large variety of different software programs. **Result:** Pupils use a software program to view different videos. Pupils use GeoGebra to draw geometric shapes.	solved using different methods. • Pupils can use ICT to research different applets. **Topic 3** • Pupils can use the Pythagorean equation to measure/calculate different sides of a right triangle. • Pupils can draw right triangles using the Pythagorean Theorem. • Pupils can draw right triangles with ICT, using the Pythagorean Theorem.		participate in a quiz about the Pythagorean Theorem.	

APPENDIX 2 – Proposed Exercises

Proposed exercises from Finland

1. Snow crystal geometry

Many artworks have been inspired by geometry of snowflakes and snow crystals. Snow crystal is a single crystal of ice, within which the water molecules are all lined up in a precise hexagonal array. The six-fold symmetry you see in a snow crystal arises from the arrangement of water molecules in the ice crystal lattice.

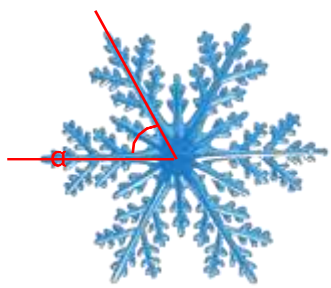
1. What is an angle α in symmetric snow crystal?

a. 30°

b. 45°

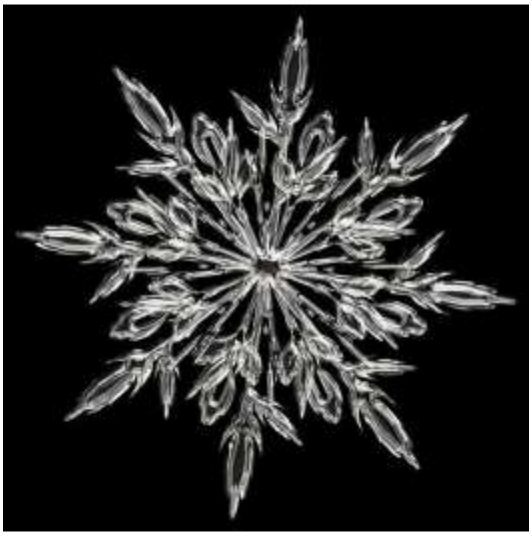
c. 60°

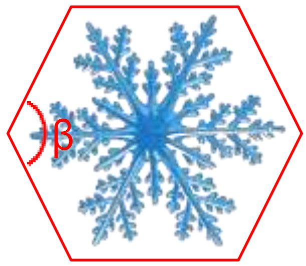
d. 75°

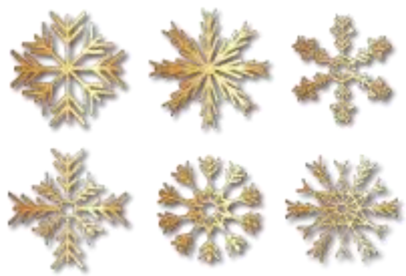


2. Every angle β in symmetric hexagonal is

a. 45°
b. 60°
c. 90°
d. 120°







2. Harmonic Series

In mathematics, the **harmonic series** is the divergent infinite series:

$$\sum_{k=1}^{\infty} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$$

Its name derives from the concept of overtones, or harmonics in music: the wavelengths of the overtones of a vibrating string are 1/2, 1/3, 1/4, etc., of the string's fundamental wavelength.

TIMBRE (=sound quality or tone colour)

You may be doing some research about important music fundamentals and have come across the term – Timbre (pronounced Tam ' bre). But what is timbre? Well, timbre is a term used to describe the different sound quality or tone colour of a particular instrument or sound source. For example, if you hear your mother talking in another room you're able to tell that it's her because of her voice's timbre. Even if your mom and sister both say the same words at the same pitch and volume, you should be able to tell who is who.

THE THEORY BEHIND TIMBRE(HARMONIC SERIES)

Sounds are created by the vibration of frequencies. Most sounds contain more than one frequency thus the additional frequencies are referred to as overtones or harmonics. These combinations or vibrations are what helps us determine what the actual sound source of a particular sound is.

The lowest frequency of a waveform is known as the fundamental frequency and is referred to as the pitch of a note in music. This frequency is the loudest. All of the other waveforms correspond to the harmonics or overtones of the sound. As you can see by the image below, the under most waveform is known as the fundamental frequency, while the waveforms above are the harmonics/overtones.

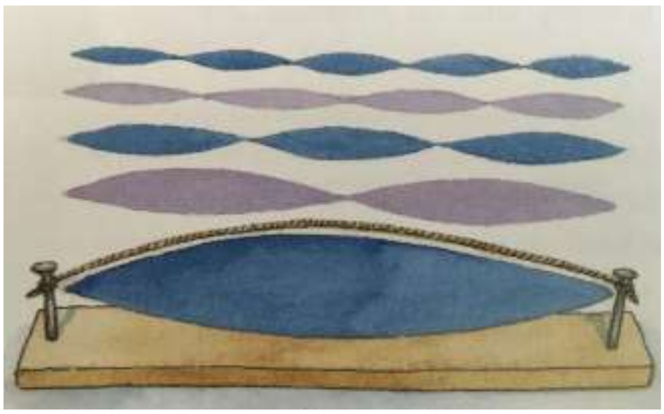
Exercise example

If the fundamental frequency is 110 Hz, the overtones frequencies are

a) 55Hz , 36Hz, 28Hz and 22Hz

b) 220Hz, 330Hz, 440Hz, 550Hz

c) 220Hz, 440Hz, 880Hz,1760Hz



Every **tone** (sustained pitch) is actually a composite of several different pitches. These 'extra' pitches, called **overtones**, are resonant frequencies occurring above the main pitch, called the **fundamental**. Most overtones are **harmonics**, which are simple fractional vibrations of the fundamental pitch (12 , 13 , 14 , and so on). Since dividing a string in half causes it to vibrate twice as fast, the frequencies of harmonics can found by multiplying the frequency of the fundamental pitch by 2, 3, 4, and so on.

Harmonic Number (n)	**Frequency in Hz**	**Pitch Name**	**Ratio from Previous Harmonic (nn-1)**	**Interval from Previous Harmonic**
1 (fundamental)	110	A	11=1.000	unison
2	220	A	21=2.000	octave
3	330	E	32=1.500	perfect fifth
4	440	A	43=1.333...	perfect fourth
5	550	C♯	54=1.250	major third
6	660	E	65=1.200	minor third
7	770	almost G	76=1.166...	small minor third
8	880	A	87=1.143	large major second

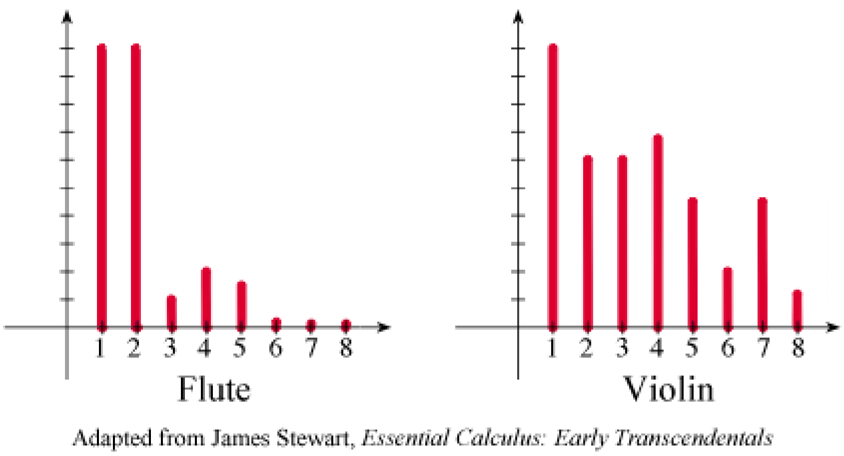
Audio examples are triangle waveforms downloaded from OnlineToneGenerator.com.

Note that the first three intervals created by successive pitches in the harmonic series are an octave, a perfect fifth, and a perfect fourth. Because of this relationship, these intervals sound particularly consonant and are called the 'perfect' intervals. The later harmonics create intervals that are gradually smaller (thirds, then seconds) and more and more dissonant.

These simple ratios became the basis of several different tuning systems, which are ways to determine the precise frequencies of named pitches.

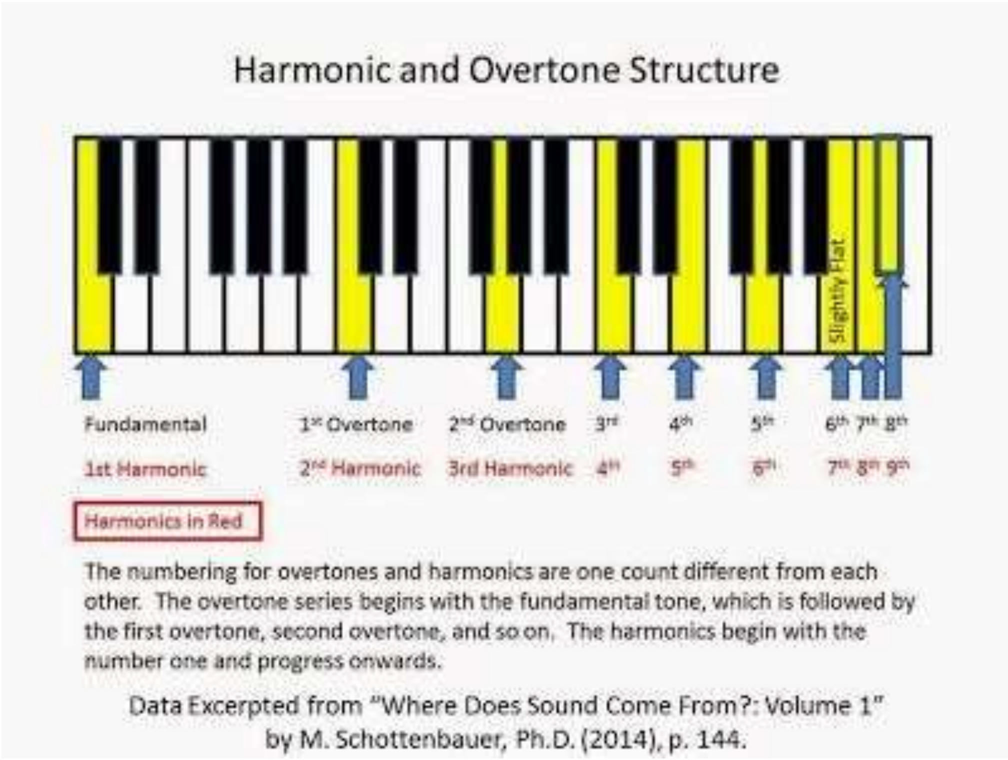
Timbre and the Harmonic Series

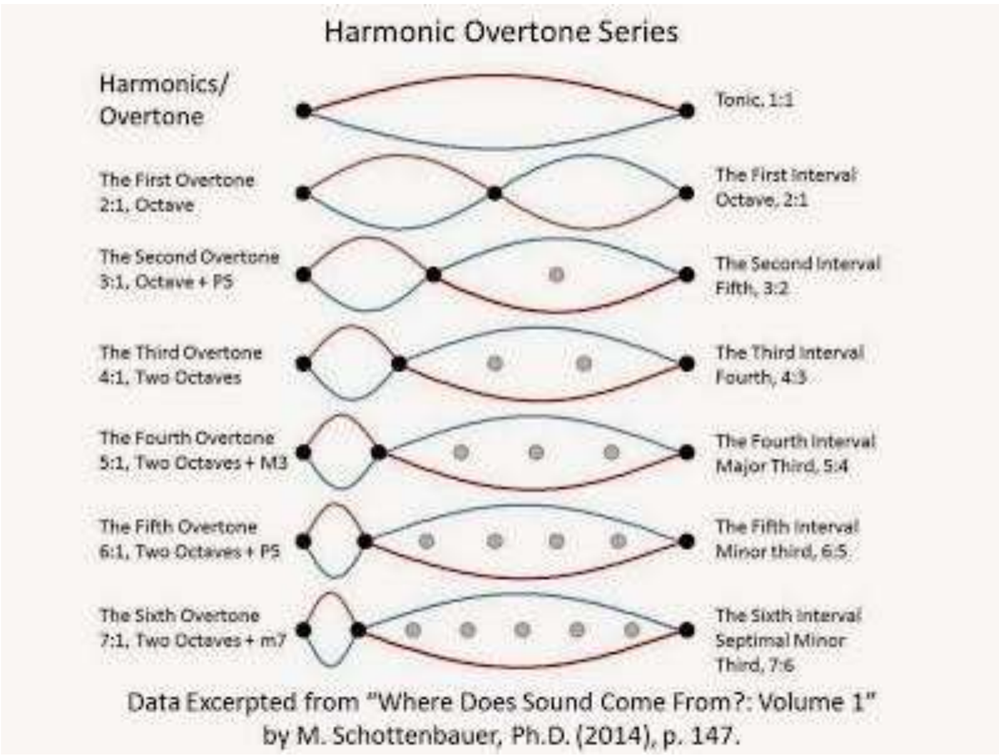
The harmonic series affects timbre because each type of instrument produces a distinctive pattern of loud and soft overtones. As we move farther from the fundamental pitch, overtones have a tendency to become quieter, but on some instruments particular overtones are more or less powerful than this tendency suggests. The graphs below are frequency power spectrums that show the strengths of the first eight harmonics of a D (294 Hz) at the same volume on a flute and on a violin.



Strengths of the first eight harmonics on a flute and a violin
(frequency power spectrums of D at 294 Hz)

The first two harmonics on the flute are very strong, but the remaining harmonics are quite weak. This is what gives the flute its characteristic pure timbre. On the violin, the second harmonic is much weaker than the flute's second harmonic, but the 3rd through 8th harmonics are much stronger than the flute's. This pattern makes the timbre of the violin sound much more complex and rich than that of the flute.


Harmonic and Overtone Structure
Slightly Flat
Fundamental
1st Overtone
2nd Overtone
3rd
4th
5th
6th 7th 8th
1st Harmonic
2nd Harmonic
3rd Harmonic
4th
5th
6th
7th 8th 9th
Harmonics in Red
The numbering for overtones and harmonics are one count different from each other. The overtone series begins with the fundamental tone, which is followed by the first overtone, second overtone, and so on. The harmonics begin with the number one and progress onwards.
Data Excerpted from "Where Does Sound Come From?: Volume 1"
by M. Schottenbauer, Ph.D. (2014), p. 144.

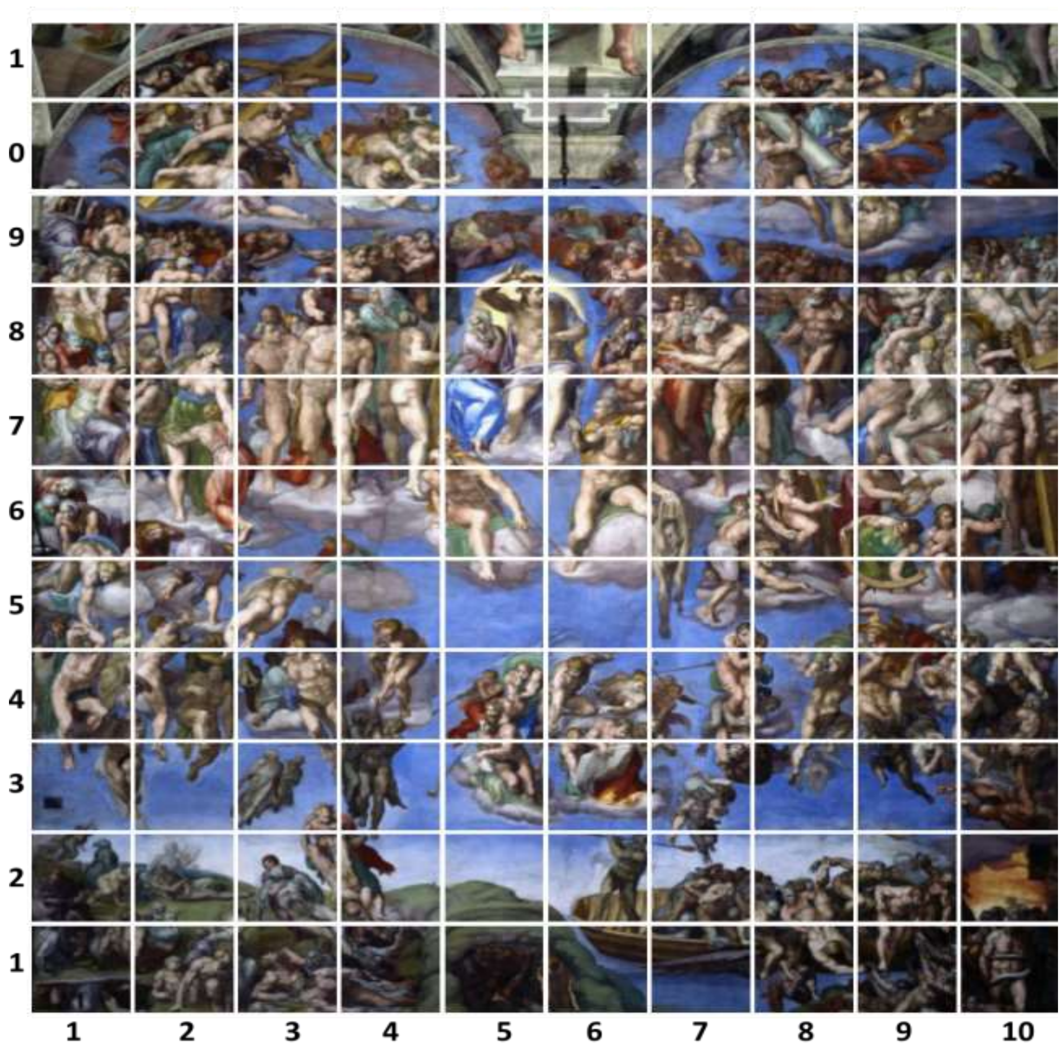

Harmonic Overtone Series
Harmonics/
Overtone
Tonic, 1:1
The First Overtone
2:1, Octave
The First Interval
Octave, 2:1
The Second Overtone
3:1, Octave + P5
The Second Interval
Fifth, 3:2
The Third Overtone
4:1, Two Octaves
The Third Interval
Fourth, 4:3
The Fourth Overtone
5:1, Two Octaves + M3
The Fourth Interval
Major Third, 5:4
The Fifth Overtone
6:1, Two Octaves + P5
The Fifth Interval
Minor third, 6:5
The Sixth Overtone
7:1, Two Octaves + m7
The Sixth Interval
Septimal Minor
Third, 7:6
Data Excerpted from "Where Does Sound Come From?: Volume 1"
by M. Schottenbauer, Ph.D. (2014), p. 147.

Proposed exercises from Italy

1. A…GIUDIZIO UNIVERSALE

In various contexts, identify, describe and construct significant relationships:

- Recognize similarities and differences.
- Recognizing in facts and phenomena relations between quantities.
- Use Cartesian coordinates, diagrams, tables to represent relationships and functions.
- Solve problems using geometric properties of the figures using deductions and appropriate representation tools
- Use different logical processes: induction and generalization, deduction, counter examples.



1. Respecting the rules of the Cartesian coordinate system's: find and write in the appropriate spaces the Coordinates of the following parts of Michelangelo's fresco.

(______;______)

(_____;_____)

(______;______)

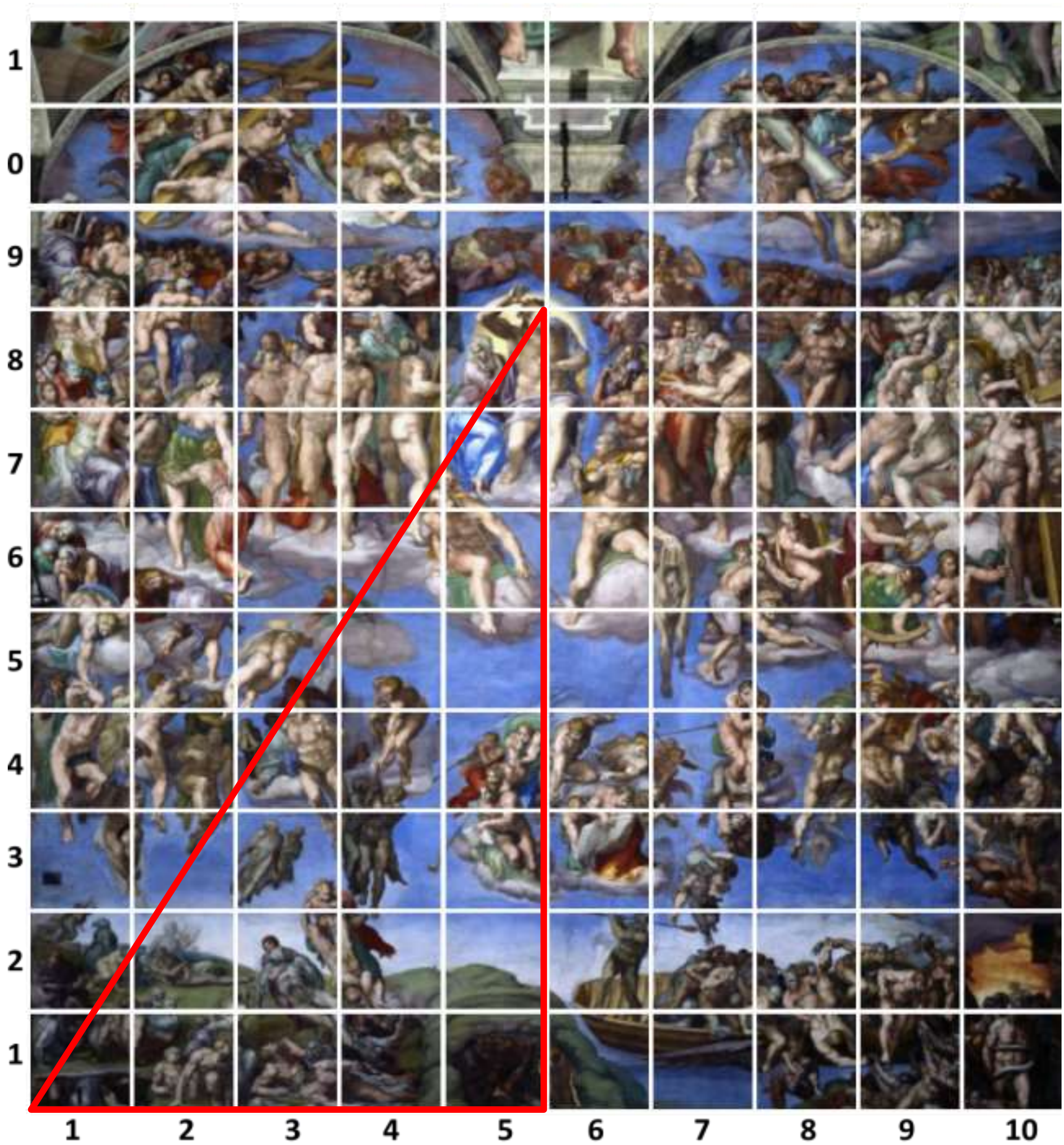
(______;_____)

2. The representation of Michelangelo's "Giudizio Universale" presented on the previous page is in 1:60 scale.

   A) How wide is the fresco in reality??

   B) How tall is the character occupying the squares that have the following coordinates: (5;7), (5;8), (5;9), (6;7), (6;8), (6;9)?

3. Calculate the real distance between the forehead of the Christ (the central point of the work – see photo on the side–) and the base vertices of the fresco.


1
0
9
8
7
6
5
4
3
2
1
1 2 3 4 5 6 7 8 9 10

2. The naumachie

- Compare and analyze geometric figures by identifying invariants and relationships
- To visualize three-dimensional objects starting from a two-dimensional representation and vice versa, to represent a solid figure on a plane
- To interpret, construct and transform formulas that contain letters to express relations and properties in general form
- Calculate lengths of circles and areas of circles
- Calculate the area and volume of the most common solid figures and give estimates of objects of everyday life
- Use the basics of plane and space geometry. Know the fundamentals of Euclidean plane and space geometry.

The naumachia (in Latin naumachia, from the ancient Greek *ναυμαχία* / naumachía, literally "Battleship"), in the age of the Roman empire, were simulations of naval battles carried out in natural or artificial basins flooded for the occasion, where they commemorate famous historical battles.

The naumacharii, that is the fighting actors, were chosen among the enemy slaves, or people hired at the time, or paid sailors or criminals condemned to death, whose life was saved if they showed skill and courage.
The Romans went crazy for naumachia, so Augusto organized several naumachie in the Septa, a monumental complex with an open space, surrounded by arcades and enriched by artworks from the conquered countries.
The almost circular basin of Augustus with a radius of 225 meters went into operation in 2 BC.
An aqueduct, about 33 km long, was created specifically to bring water from the lake of Martignano.

The aqueduct, which had a flow of 180 liters of water per second, was used to fill the basin of Augustus with water for 1.5 meters in height. This depth was sufficient to allow ships to float and not let them touch the bottom.

How many days was it necessary to fill the basin so that the naumachia took place?

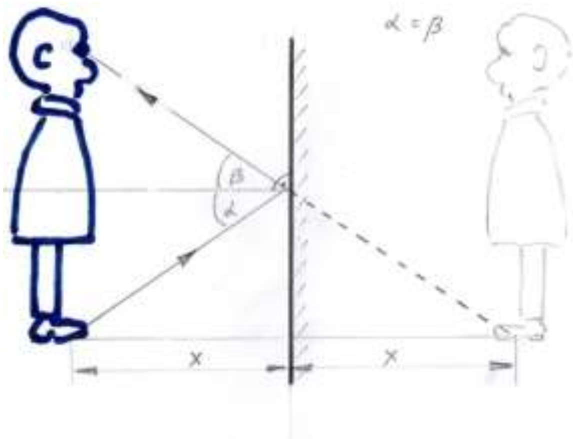
If the candidate does not find the correct result of the question, he will recycle all the numbers and mathematical symbols used to create an artistic work like *Tobia Ravà*.

Proposed exercises from Estonia

1. The Plane Mirror - Science II

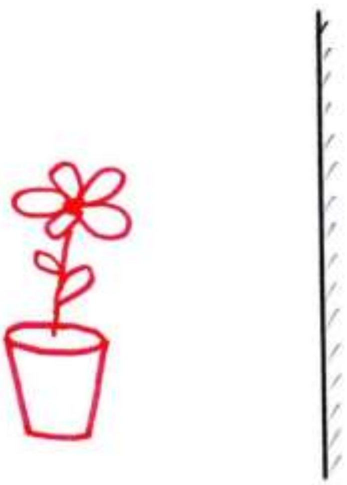
I THEORY – IMAGE FORMATION IN PLANE MIRRORS

The distance of the image to the mirror is equal to the distance of the object to the mirror.



II TASKS

1. Draw the image of an object in a plane mirror.

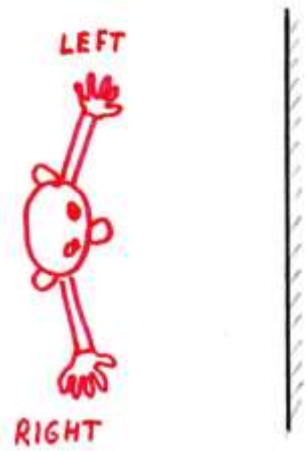


2. Is the image of the object upside down compared to the original object?

3. Look in the mirror. Wave your right hand. Which hand does the reflection use to wave back?

4. Why the image is right-left reversed?

5. Draw the top view of your reflected image



6. Is the image is right-left reversed in your drawing?

III PRACTICAL PART

Test equipment: 2 or more plane mirrors

1. Would it be possible to reflect an object so that the left and right side were not reversed?

In order to find the answer, it would be necessary to test the mirrors To carry out a test successfully you need at least 2 mirrors.

Draw a diagram to show the position of mirrors so that the image is not left-right reversed. Use the analogy with the first part of the activity and draw the top view.

2. Would it be possible to reflect an object so that the reflected image was upside down?

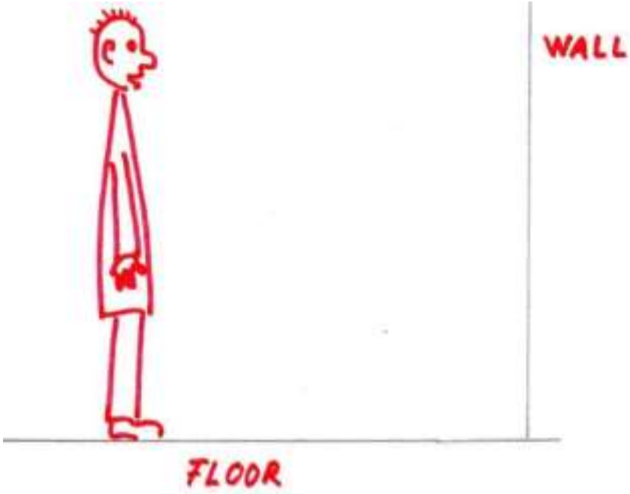
To carry out a test successfully you need at least 2 mirrors.

Draw a diagram to show the position of mirrors so that the image is not upside down. Use the analogy with the first part of the activity and draw the side view.

IV APPLICATION OF THE THEORETICAL PART

1. What should be the minimum height of a mirror so that a person could see the reflected image from top to toe? How high should the mirror be fitted?

To do this you have a person, a wall, and the floor. Fix the mirror in the right height and as small as possible.



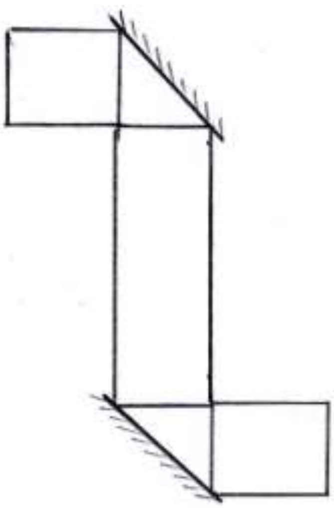
V PHYSICS AND ART

Test equipment: mirrors, plasticine or glue, paper or cardboard for tubes (pieces of PVC pipe or something similar)

Task: build a device that helps to look round the corner, over or under the fence. Draw a figure to explain the operation principle of the instrument.

APPENDIX

A possible version of periscope:



2. Ant-Man and Science

"Ant-Man (Figure 7) is a 2015 American superhero film based on the Marvel Comics characters of the same name: Scott Lang and Hank Pym. After trying the suit on, Lang accidentally shrinks himself to the size of an insect[2]. Armed with a super-suit with the astonishing ability to shrink in scale but increase in strength, cat burglar Scott Lang must embrace his inner hero and help his mentor, Dr. Hank Pym, plan and pull off a heist that will save the world.[3]"

Figure 7. Ant-Man
(https://goo.gl/images/RX1Hq c)

Solving the tasks on this hand-out you will get the answer to the question "Is it possible to enlarge/shrink a person the way you wish? What does science say about it? "

While solving the task it is allowed to use the Internet to find the information required.

Part I. Prior knowledge

Recall:

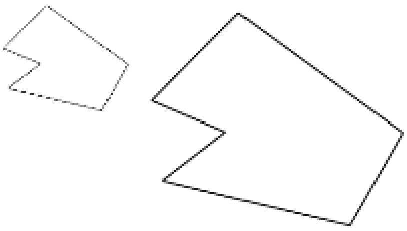


Figure 8. Geometric figure

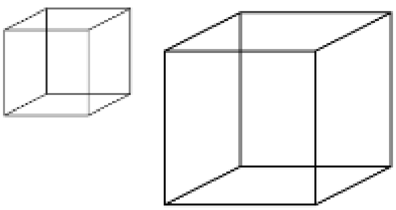


Figure 9. Solid object

[2] Ant-Man (film). (2019). In Wikipedia. Retrieved February, 13, 2019, from https://en.wikipedia.org/wiki/Ant-Man_(film)

[3] IMDB. (n.d.). *Ant-Man (2015*. Retrieved from https://www.imdb.com/title/tt0478970/

1. How does the area of a solid object depend on linear measurements (*the linear measurements are height, length and width)*?

....

2. How does the volume of a solid object depend on linear measurements?

....

3. How does the perimeter of a geometric figure change when its linear measurements are increased k times and the proportions are left unchanged?

....

4. How does the area of a geometric figure change when its every side will be enlarged k times?

....

5. How does the volume of the object change when its every linear measurement is enlarged k times?

....

6. Draw the cross-section of the cuboid (Figure 10)

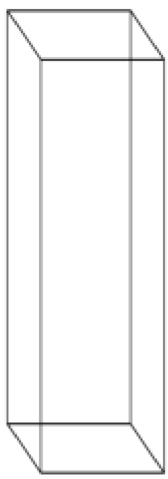


Figure 10. Cuboid

7. What does the mass of an object depend on?

……

8. How does the weight of a body change when its measurements are increased k times when its density remains the same?

……

9. What does the heat produced by the body depend on - on its area or volume?

……

10. Does the heat loss through the surface of the body depend on the linear measurements of the body or its area?

……

11. What is muscular strength? What does muscular strength depend on - on the length of the muscle, the area of its cross-section or mass?

……

12. How does muscular strength change when we increase the linear measurements k times?

……

13. The consumption of oxygen is related to the energy produced by the body. Does the consumption of oxygen depend on the area or volume?

……

Part II. From an ordinary man to Ant-Man

Let's presume that we want to turn an ordinary man into a superhero with the size of an ant reducing his height 100 times.

Draw the man's figure and add the linear measurements that correspond to an average man.	Draw an ant and add its linear measurements.

Part III. Physics vs biology

1. **An eagle and a bumblebee (How does the ability to fly depend on the bodily structure?)**

Draw an eagle	Draw a bumblebee

2. What is the size of an eagle's wings and a bee's wings compared to the size of their body?

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…….………………………………………………………………………………….………………………………………………………………

…….………………………………………………………………………………….………………………………………………………………

NB! The lift force depends on the wing area. The mass depends on the volume. If we reduce the eagle to the size of the bee, then the reduction in the amount of lift equals the square of the linear measurements. The weight changes as the cube of the linear measurements.

3. What happens to a bumble- bee's ability to fly when it is changed to the size of an eagle (for example, to enlarge it 100 times)?

……………………………………………………………………………………………..………………………………………………………………

…….………………………………………………………………………………….………………………………………………………………

…….………………………………………………………………………………….………………………………………………………………

4. The "Bumble-bee paradox" is often talked about, as with such a body structure, it is impossible for a bumblebee to fly. What exactly is this paradox about and how can their ability to fly be explained?

……………………………………………………………………………………………..………………………………………………………………

…….………………………………………………………………………………….………………………………………………………………

…….………………………………………………………………………………….………………………………………………………………

5. What is the system bumblebee's use for breathing called?

……………………………………………………………………………………………..………………………………………………………………

…….………………………………………………………………………………….………………………………………………………………

…….………………………………………………………………………………….………………………………………………………………

6. Will a bee suffocate when to turn it into the size of a dog? (Compare the use of energy and ability to consume oxygen when the bee is changed to the size of a dog.)

……………………………………………………………………………………………..………………………………………………………………

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7. **Ancient insects were huge (see Figure 11). How could it be explained that no such insects exist nowadays?**

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…….…………………………………………………………………………………….…………………………………………………………….

…….…………………………………………………………………………………….…………………………………………………………….

Figure 11. https://nauka.boltai.com/wp-content/uploads/sites/26/2017/02/023.jpg

Big whale/small whale

"The **humpback whale** (*Megaptera novaeangliae*) is a species of baleen whale. One of the larger rorqual species, adults could be about ………… m long. Found in oceans and seas around the world, humpback whales typically migrate up to 25,000 km (16,000 mi) each year. They feed in polar waters, and migrate to tropical or subtropical waters to breed and give birth, fasting and living off their fat reserves."[4]

[4] Humpback whale. (2019). In Wikipedia. Retrieved February, 13, 2019, from https://en.wikipedia.org/wiki/Humpback_whale

8. The length of the whale is missing in the previous article. Study the Figure 12 and give the estimate of the length of an average humpback whale.

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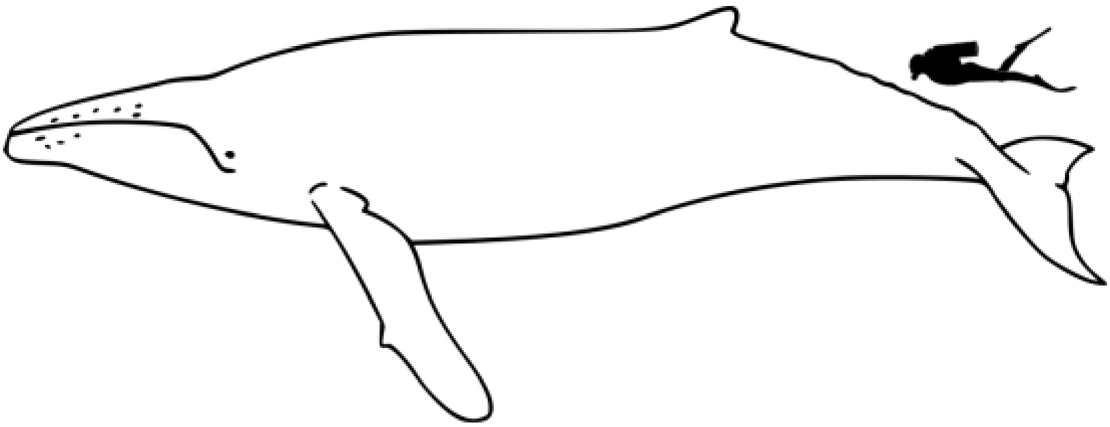


Figure 12. [*https://en.wikipedia.org/wiki/Humpback_whale#/media/File:Humpback_whale_size.svg*](https://en.wikipedia.org/wiki/Humpback_whale#/media/File:Humpback_whale_size.svg)

9. The ratio between a grown-up whale and a newborn calf is 1:4. If the calf is four times smaller, its ability to retain body heat is …………………smaller/bigger and ability to generate body heat is …………………. smaller/bigger.

10. The humpback whale can easily live in cold waters. What enables that?"

………..

………..

………..

………..

11. Why do humpback whales feed in the Arctic waters and cold waters surrounding Antarctica, but come to give birth in warm waters where there is not enough food for them?

………..

………..

……………………………………………………………………………………………………..
……………………………………………………………………………………………………..

12. What happens to the whale when its measurements turn into the ones of a shrewmouse? What is the biggest danger for him?

……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..

Part IV. Let's play with the figures: Ant-man, really?

What happens when we make a person 100 bigger or smaller (3 measurements: height, length and width at the same time)?

1. What happens to the mass of a person when to reduce their size 100 times?

……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..

2. What happens to a person's volume when to enlarge the body 100 times?

……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..
……………………………………………………………………………………………………..

3. How many times less material is needed when we reduce the size of the body 100 times?

…………………………………………………………………………………………………..
…………………………………………………………………………………………………..
…………………………………………………………………………………………………..
…………………………………………………………………………………………………..

4. **When the height was reduced 100 times, how much was the reduction in the muscular strength?**

…………………………………………………………………………………………………..
…………………………………………………………………………………………………..
…………………………………………………………………………………………………..
…………………………………………………………………………………………………..

5. **When the height is reduced 100 times, how many times will the mass decrease?**

……………………………………………………………………………………..

6. **We make the person 100 smaller. Before that, the person could lift an object equal to their weight. How many times heavier an object than their body can they lift now?**

…………………………………………………………………………………………………..
…………………………………………………………………………………………………..

7. **How does the heat loss change when we reduce the body 100 times?**

…………………………………………………………………………………………………..
…………………………………………………………………………………………………..

8. **How will the ability to produce heat change when the body is reduced 100 times?**

…………………………………………………………………………………………………..
…………………………………………………………………………………………………..

Part V. Summary

1. **Is it possible to use the facts from physics and biology to create such a situation that a person could be reduced to the size of an ant? What dangers could be waiting for that person? Write your opinion**

……..
……..
……..
……..
……..
……..
……..
……..

2. **Design an Ant-Man Action Figure Model (see Figure 13) of yourself (1:10).**

Figure 13. Ant-Man Action Figure Model (https://flic.kr/p/MiZ2Gi)

In order to do this:

a) Take your main body measurements. Reduce them 10 times. If necessary, add some more necessary measurements. Think about how to round results.

Body part	Actual size (cm)	10 times reduced (cm)
height		
arm length		
head height		
leg length		
waist size		
foot length		

b) Model your personal Ant-Man Action Figure Model from plasticine.

***3*. Would you like to be the size of an ant? Why?**

....

....

....

....

....

....

....

....

3. Mondrian Art

Mondrian Art is the style of painting popularized by **Dutch artist Piet Mondrian** during the early 1900's. In addition to influencing art, the Mondrian look has shown up in **fashion**, **architecture**, **advertising**, **design**, and more[5].

What you will need:

- a pencil
- a ruler
- a black marker with a wide tip
- white, red, blue and yellow paper sheets
- white art paper
- glue

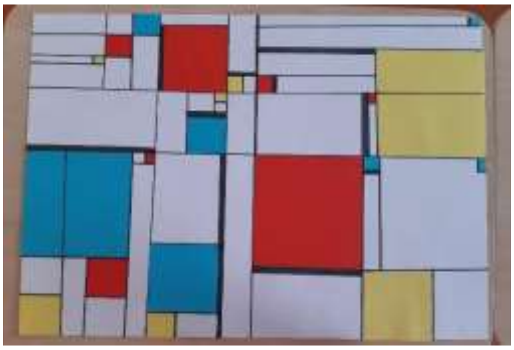
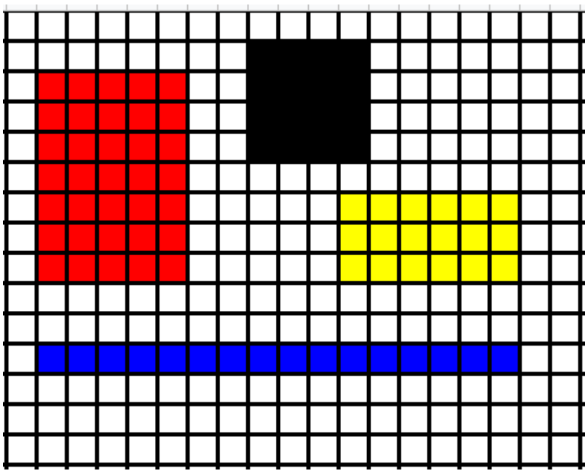


Fig.14: Original artwork inspired by Mondrian

I The area and perimeter of a rectangle

1. Which rectangle has a bigger/smaller perimeter? Give a visual estimate.



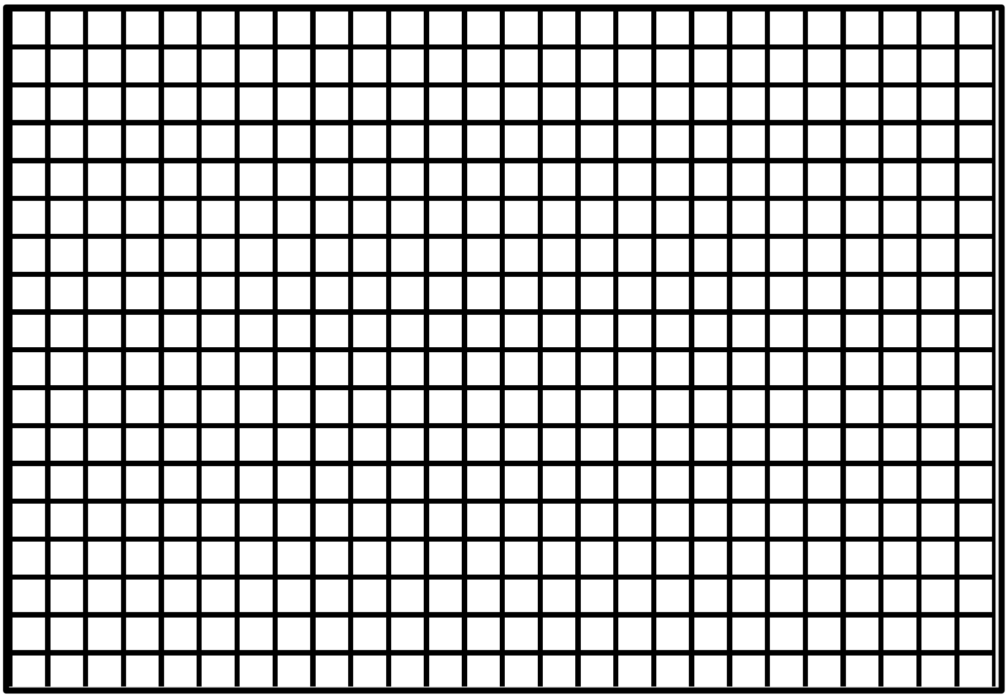
[5] Contorl Alt Achieve. (2018) *Make Mondrian Art with Google Sheets*. Retrieved from https://www.controlaltachieve.com/2018/10/mondrian-sheets.html

2. Write as many expressions as possible to calculate the perimeter of each rectangle. (The length of each side of the square is 1 unit.)

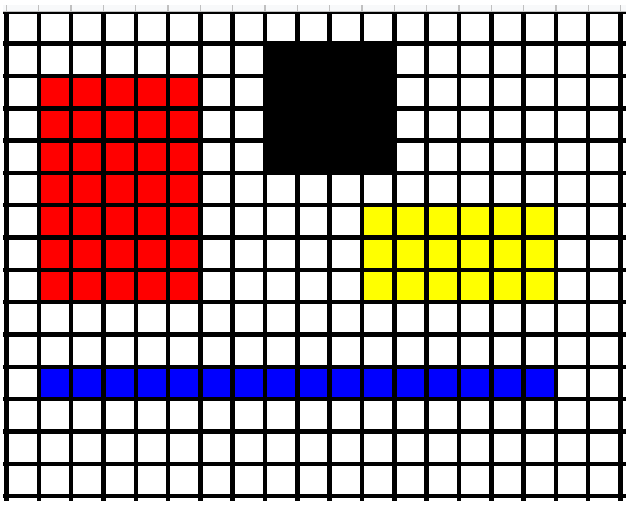
Red	Blue	Yellow	Black

3. Which rectangle has the biggest/smallest perimeter?

4. Draw as many rectangles as possible the perimeter of which is 28 units.



5. Which rectangle has the biggest/smallest area? Give a visual estimate.

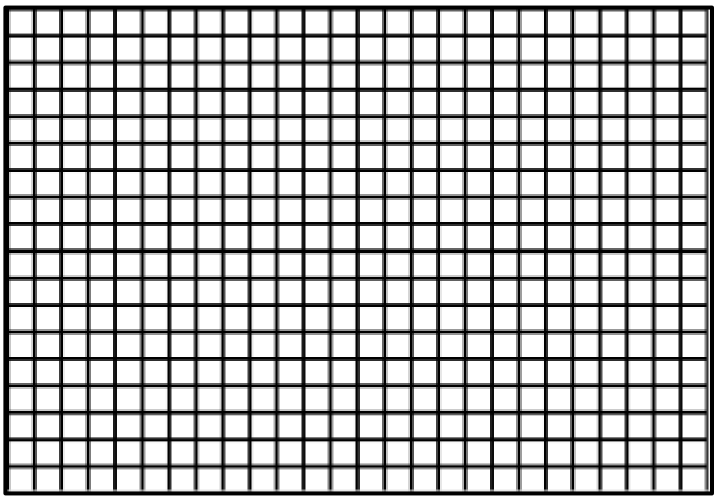


6. Calculate the area of each coloured rectangle.

Red	Blue	Yellow	Black

7. Study the rectangles you drew in question 3. Calculate the area of each rectangle. What are the measurements of the rectangle the area of which is the largest?

8. Draw as many rectangles as you can with the area of 28 units each.



II The Golden Ratio and Golden Rectangle

The Golden Ratio, often referred to as Phi, is one of the most ubiquitous irrational numbers known to man. The properties of the Golden Ratio are found if a line is divided into two segments in such a way that the ratio of the total length to the length of the longer segment is the same as the ratio of the length of the longer segment to the length of the shorter segment. Numerically, this ratio equals $(1+\sqrt{5})/2 \approx 1.618$2[6].

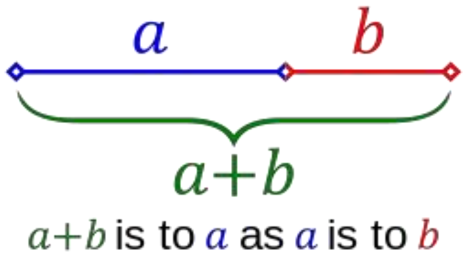


Figure 15. Golden ration line.
(https://upload.wikimedia.org/wikipedia/commons/thumb/4/44/Golden_ratio_line.svg/270px-Golden_ratio_line.svg.png)

The works of Piet Mondrian, a Dutch painter of the early 20th century, have been scrutinized for these reasons pertaining to the Golden Ratio. Mondrian heavily contributed to the De Stijl ("The Style") art movement and created a non-representational form, which he termed NeoPlasticism. His works are primarily composed of simple geometric shapes, primarily rectangles, and primary colors. The Golden Rectangle often appears in his paintings; the height-to-width ratios of specific rectangles approximately equal 1.618. It is true that Mondrian's works exhibit rectangular figures with ratios near the Golden Ratio. However, the debate as to whether or not the artist considered the ratio in any conscious way is left unsettled, and provides a context for a mathematical analysis for an idea that is seemingly not mathematically testable.

In geometry, a golden rectangle is a rectangle whose side lengths are in the golden ratio is approximately 1.618.

[6] Murphy Judy. (2017). Math and Mondrian. Retrieved from https://www.math.upenn.edu/~ted/210S17/Projects/Sample/Paper4.pdf

Golden rectangles have the property of self-similarity when adding or removing similar golden rectangles[7].

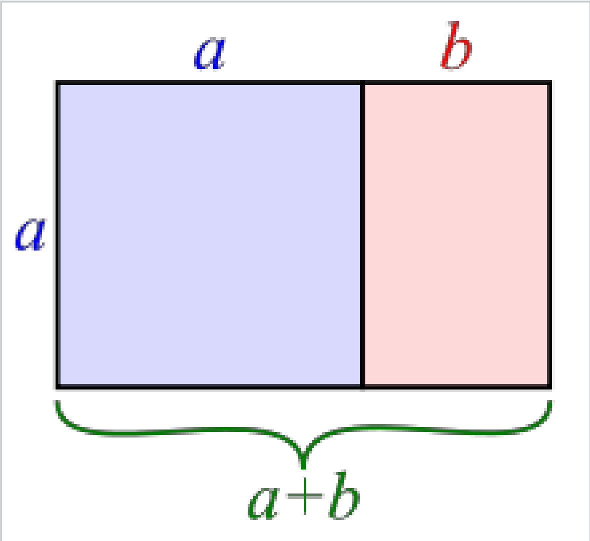


Figure 16. Similar Golden Rectangles (https://en.wikipedia.org/wiki/File:SimilarGoldenRectangles.svg)

1. **Look at a detail of Piet Mondrian's, Place de la Concorde, 1938-1943. Choose 3 different rectangles and check if the ratio of the sides allows it to call them golden rectangles.**

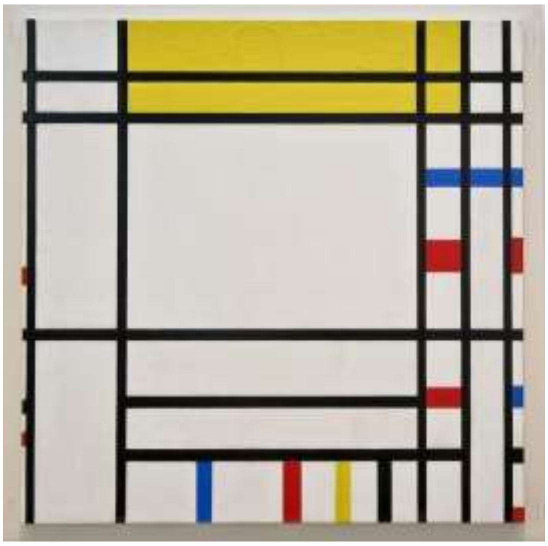


Figure 17. Sharon Mollerus. Piet Mondrian, Place de la Concorde, 1938-1943 (https://www.flickr.com/photos/clairity/12548871095/in/photostream/)

[7] Golden rectangle (2019). In Wikipedia. Retrieved February, 17, 2019, from https://en.wikipedia.org/wiki/Golden_rectangle

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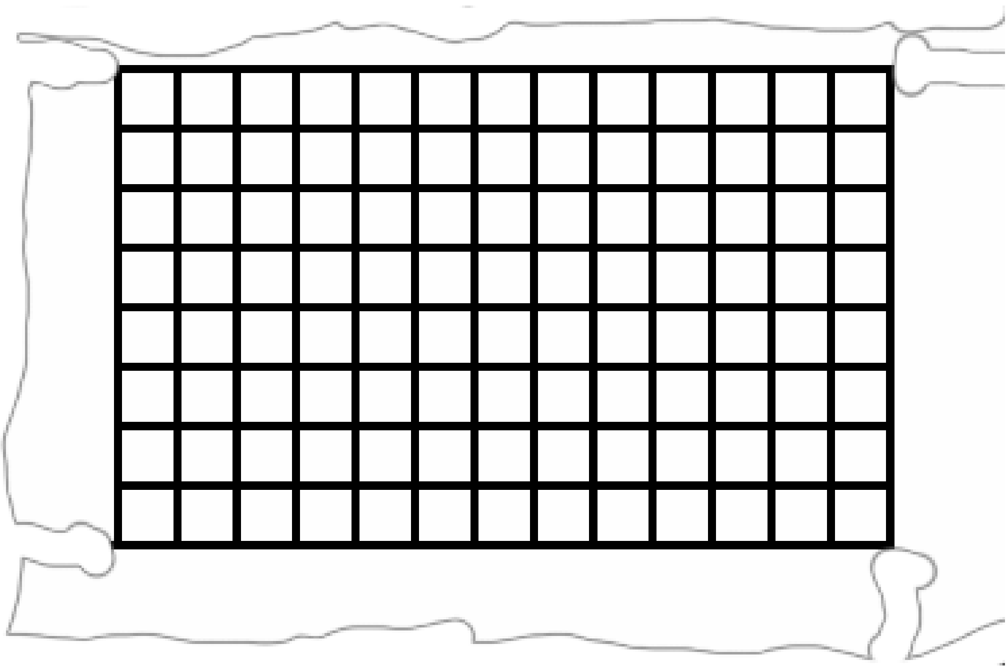
2. You noticed that the Golden Rectangle consists of a square and a rectangle. Divide the given rectangle into as many squares as possible so that each next square has only one side in common with the previous one. Number the squares that you draw.



3. Write the length and the area of each of the square you got.

Number of the square							
Side							
Area							

III Fibonacci Sequence

There is a special relationship between the Golden Ratio and the Fibonacci Sequence:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ...

(The next number is found by adding up the two numbers before it.)

And here is a surprise: when we take any two successive *(one after the other)* Fibonacci Numbers, **their ratio is very close to the Golden Ratio**[8]**.**

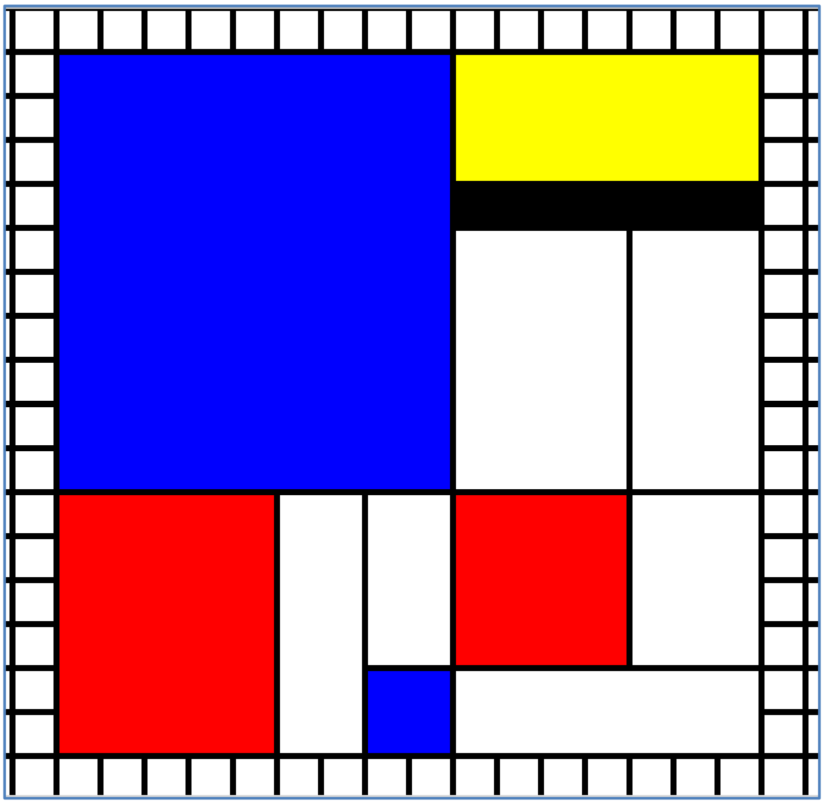
1. **Use a calculator and find the ratio of the two consecutive Fibonacci Numbers. What did you notice?**

2 : 1	
3 : 2	
5 : 3	
8 : 5	
13 : 8	
21 : 13	
34 : 21	

[8] Math is Fun. (n.d.). Golden Ratio. Retrieved from https://www.mathsisfun.com/numbers/golden- ratio.html

IV Mondrian Art

1. **Study the picture and answer the following questions:**



a) **What is the area covered by each colour?**

Blue	Red	Yellow	White	Black

b) **What is the percentage of the total area covered by each colour?**

Blue	Red	Yellow	White	Black

c) **What is the percentage of each coloured area of the area covered in blue?**

Blue	Red	Yellow	White	Black
100%				

2. **Draw a golden rectangle on a white sheet of paper and make it as large as (shorter side - 15 cm and the ratio of the sides – 1:1.618).**

3. **Calculate the length of the longer side.**

4. **Divide the rectangle into a square and a rectangle so that the ratio of the rectangle's sides is the Golden Ratio.**
5. **Repeat it on red, yellow and blue coloured paper.**
6. **Draw three horizontal and three vertical lines on the white art paper.**
7. **Cut out all the squares of different size and place them on an A3 sheet so that the picture has a balance and looks of complete.**

Follow one of these options:

1) 50% of the area of the sheet is covered in colours.
2) The area covered in one colour forms 50% of the area covered in the other colour.
3) The area covered in every next colour forms 50% of the area covered in the previous colour.

Appendix 1

Example - how to divide a color sheet of paper into Golden Rectangles.

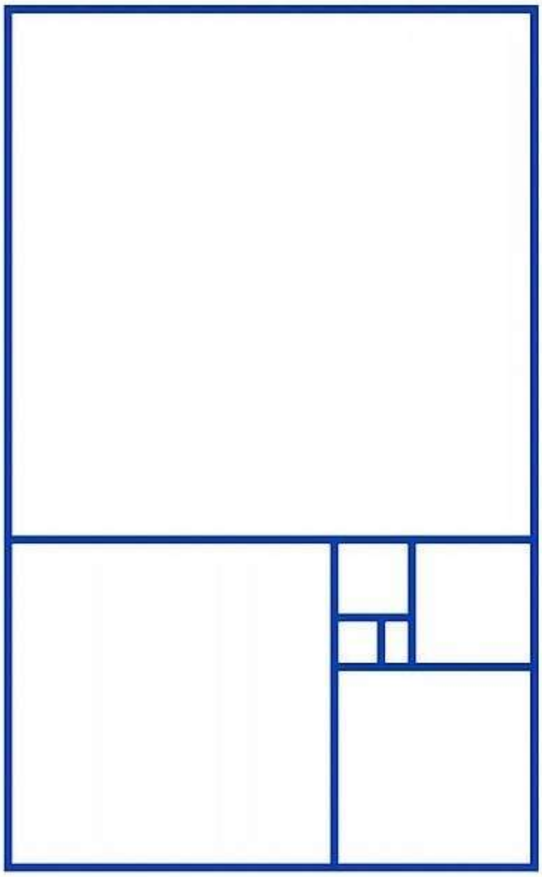
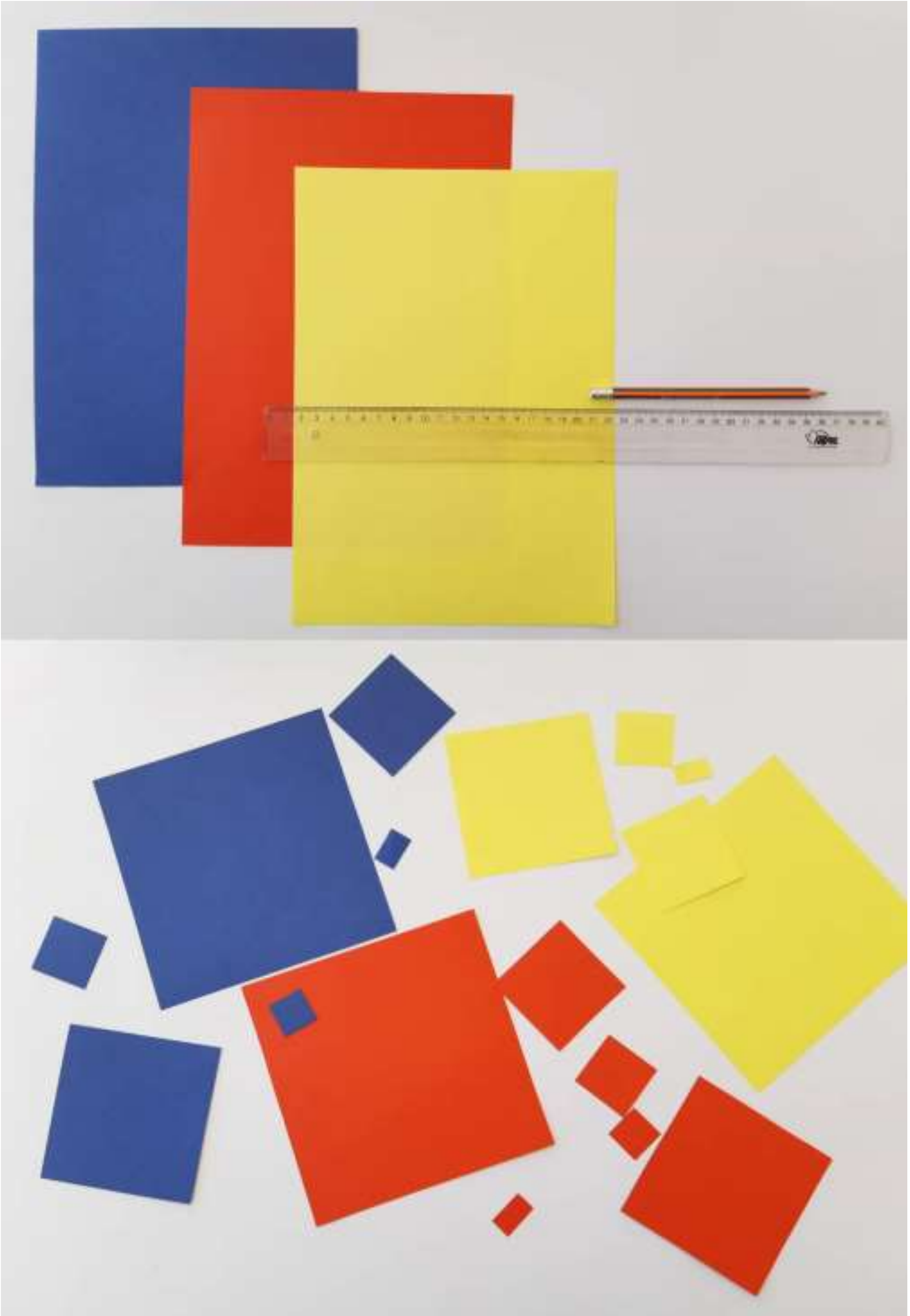
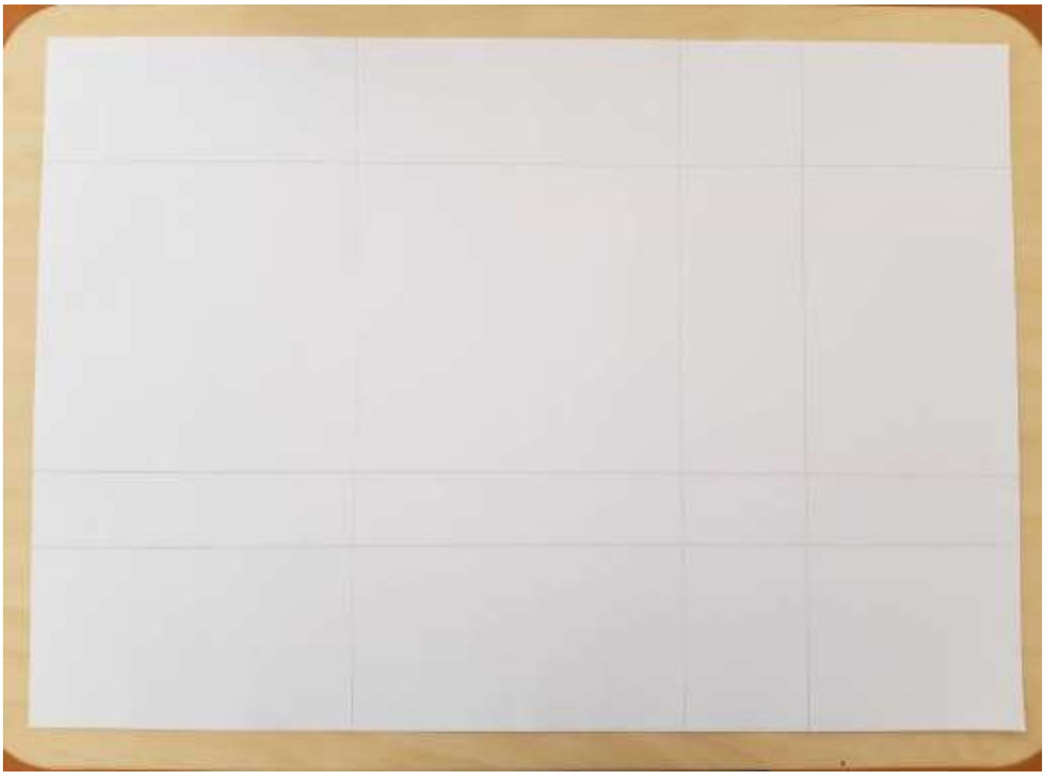


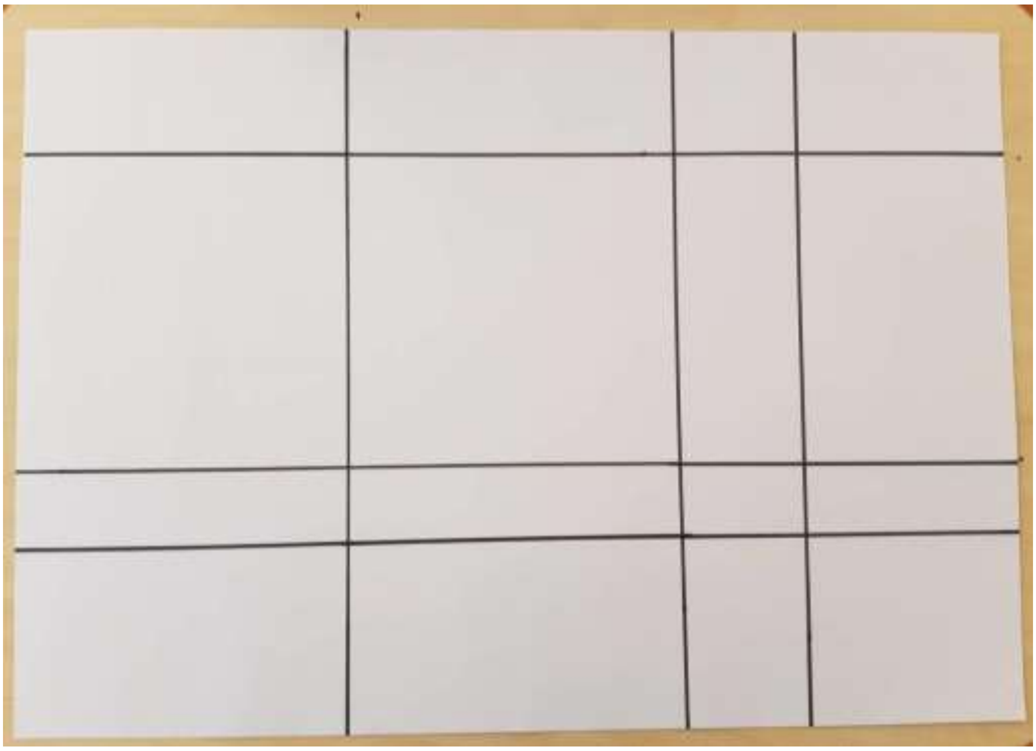
Figure 18. Golden Rectangle Ratio
(https://emptyeasel.com/wp-content/uploads/2009/01/golden-rectangle- ratio.jpg)

Appendix 2



Appendix 3





4. Estonian Ornament

Folk costume

Earlier data about clothes date from the 11th–13th century archaeological material. The chief items of female clothing were a linen shirt with sleeves and a woollen shirt-like coat. A woollen wrap-skirt was wrapped round the hips and fastened with a belt. This type of clothing survived until the 19th century. Amongst items of clothing, belts and mittens were believed to have the most protective powers[9].

Figure 19. Belts and mittens.
(http://www.estonica.org/missingimage/?filename=909285460313- vood_type_1024x1080.jpg)

Ornament

An ornament (pattern, décor, design) is a decoration made up of geometric shapes and parts of them, sometimes designed using signs and symbols, found on clothing, buildings, artefacts and elsewhere formed during the manufacturing process.. Estonian folk costumes are very beautiful and colorful. **The geometric ornament** had both aesthetic and magical significance. The common patterns were the thuja, the circle, the wheel cross, the spear wheel, the eight-pointed star, the cross, the three- or five-branched twig, the flower of the thuja, the double cross and the rose. The eight arms of the eight-headed (orthogonal) cross symbolize points of the compass. This lucky star protects our property, protects everything. It is also the starter of a new day or new happiness[10].

[9] Estonika. (2012). *Folk costume*. Retrieved from http://www.estonica.org/en/Culture/Traditional_folk_culture/Folk_costume/
[10] Eesti rahvakunst käelises tegevuses koolieelses eas. (n.a.) *Ornament*. Retrieved from http://www.tlu.ee/opmat/tp/eesti_rahvakunst/ornament.html

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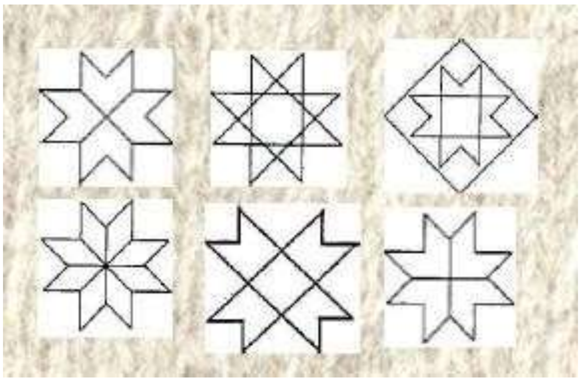
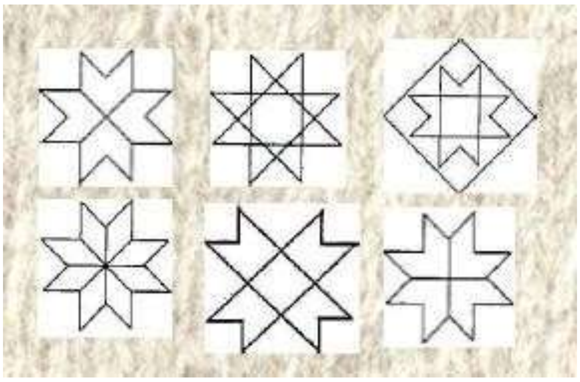
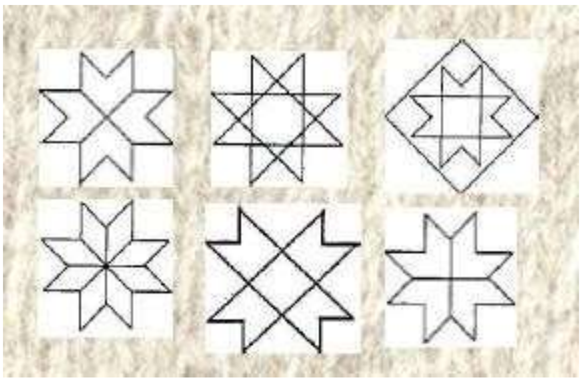
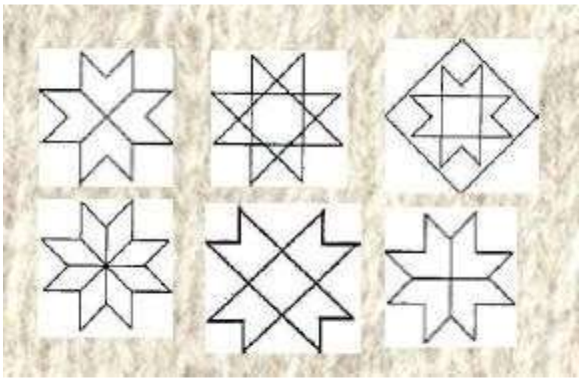
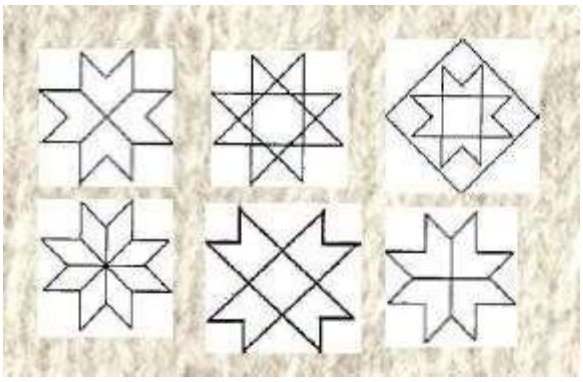
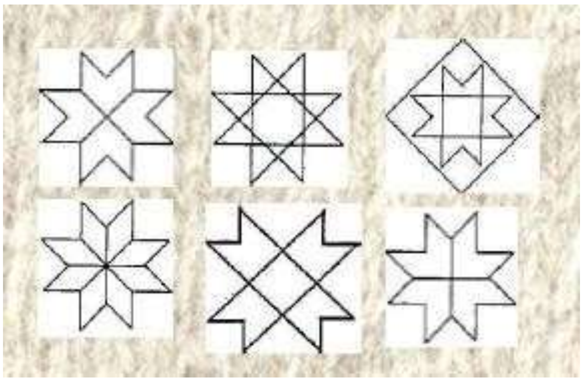
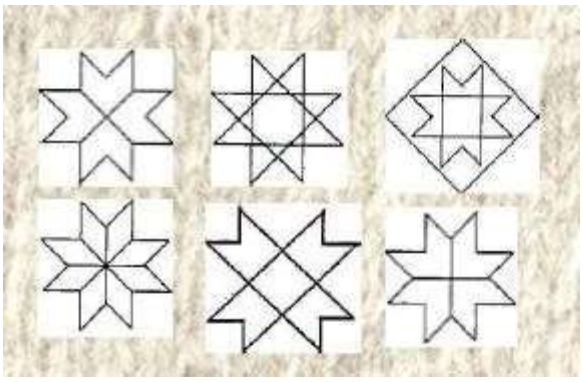
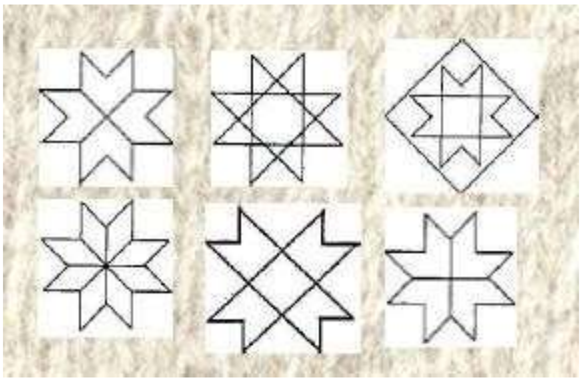
In today’s lesson, we are going to study ornaments in Estonian folk art and create geometric ornaments using our knowledge of geometry.

Part I. Geometry in the pattern

1. **Colour the corresponding geometric shape in the pattern element.**

Geometric shape	Pattern element
triangle	 https://wi.ee/wp-content/uploads/2015/10/Presentation-KIRI.pdf
square	
rectangle	

parallelogram	
trapezoid	
pentagon	
hexagon	

octagon	

2. **Write the list the names of all the geometric shapes you can see in the surface distribution net-scheme on the belt band of Vändra[11].**

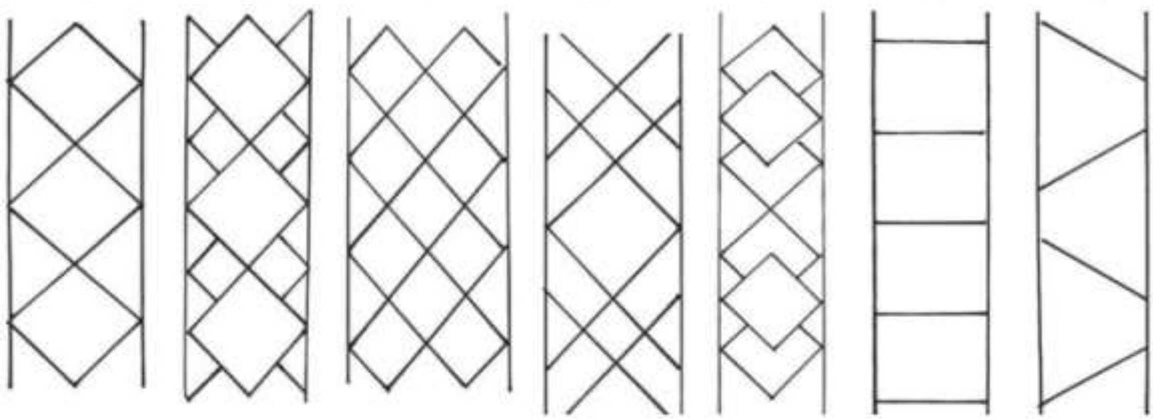


Figure 20. Net-scheme on the belt bänd.

(http://muuseum.viljandimaa.ee/aastaraamat/2010_toim_1/raud_kirivood.pdf

[11] **Vändra** is a borough in Põhja-Pärnumaa Parish in Pärnu County, Estonia. Vändra is the birthplace of the Estonian poet Lydia Koidula (1843–1886). (https://en.wikipedia.org/wiki/V%C3%A4ndra)

3. **Check, what Estonian belt bands look like:**

http://www.rahvaroivad.folkart.ee/rahvaroivad?element%5Bsugu%5D=naine&element%5Btyyp%5D=V%C3%B6%C3%B6d&element%5Bese%5D=&search=1#22

Use the search options (e.g. choose men's or women's belts)

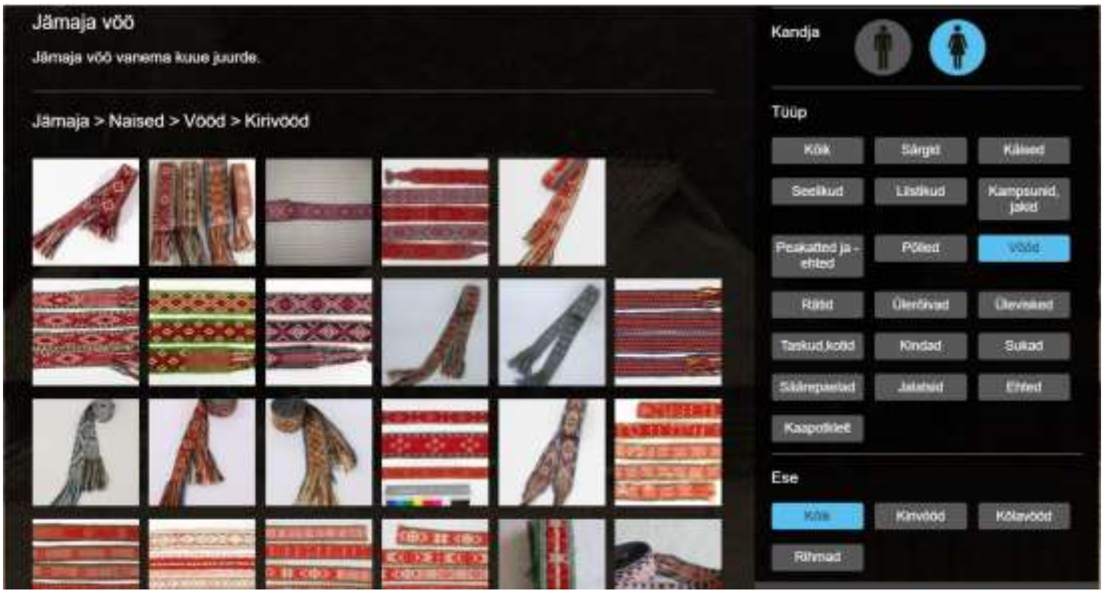


Figure 21. Belt bands on www.rahvaroivad.folkart.ee

4. **What are the colours Estonians use in belt patterns?**

5. **Using different geometric shapes design your own belt ornament. (Do not be restricted to squares only!)**

Choose one of the options

I. On paper: https://incompetech.com/graphpaper/ (choose the template you need and create a pdf file for printing it out)

II. Online: https://grid-paint.com/ (requires an account!) (Pict. 3)

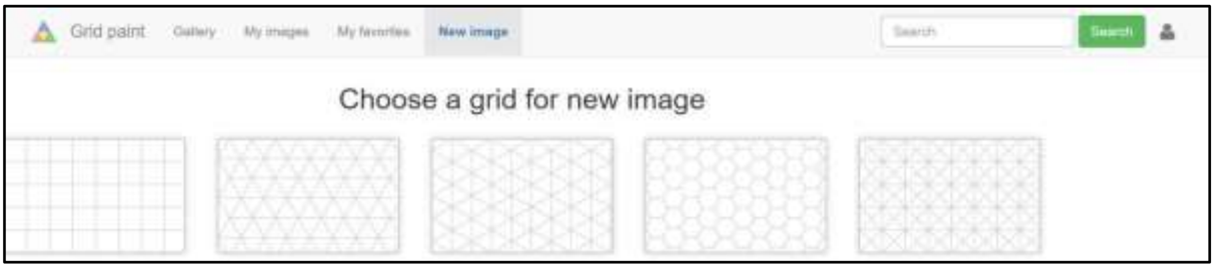


Figure 22. Grid paint

Part II. GeoGebra

Designing belt ornaments in GeoGebra

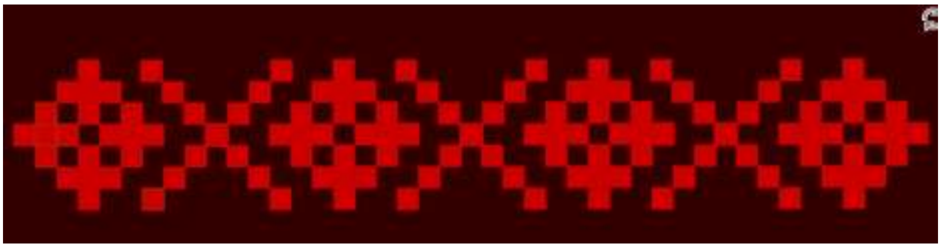


Figure 23. Belt ornament. (http://archive.novator.team/attach/42750)

1. **Open new fail in http://web.geogebra.org, choose Apps: Geometry. (Figure 24) Turn in Grid. (Figure 25)**

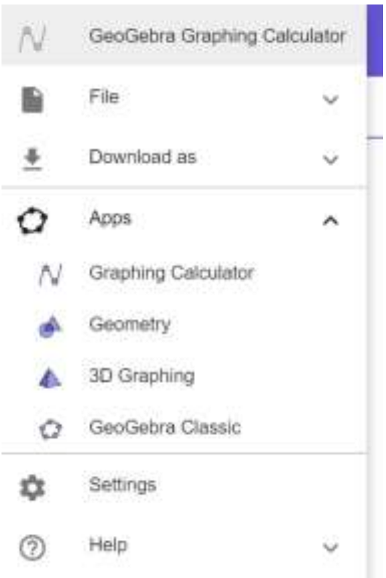


Figure 25. Apps

Figure 24. Show Grid

2. **Create the main element – the cross. It consists of five squares. Start from one square (tool: Regular Polygon) (Figure 26), colour it red (mouse right click: Setting) (Figure 27)**

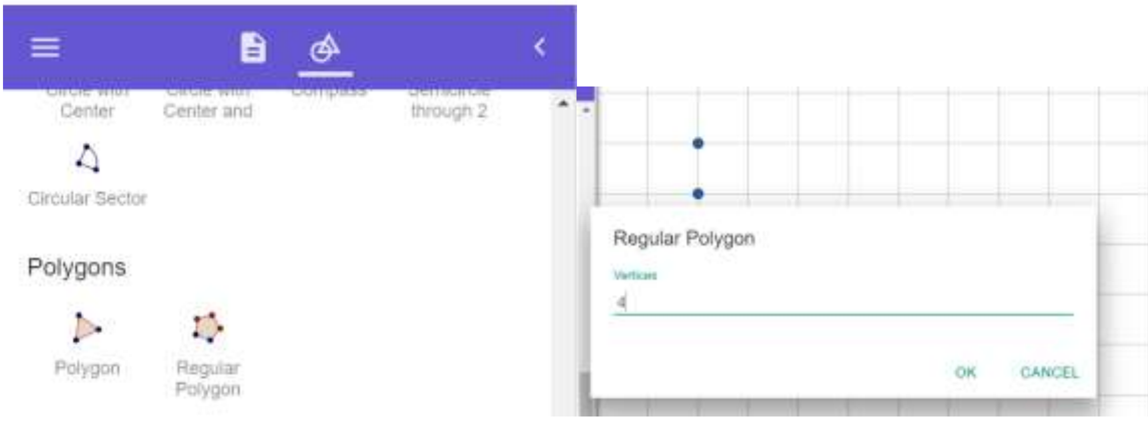


Figure 26. Regular Polygon -> Vertices 4

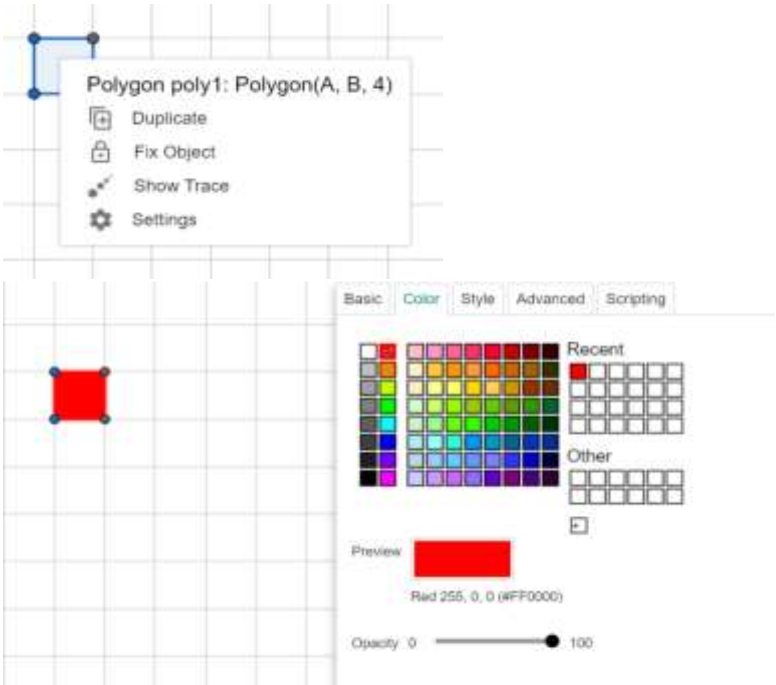


Figure 27. Settings -> Color

3. **Create other squares too, using the previous steps or duplication. (Figure 28)**

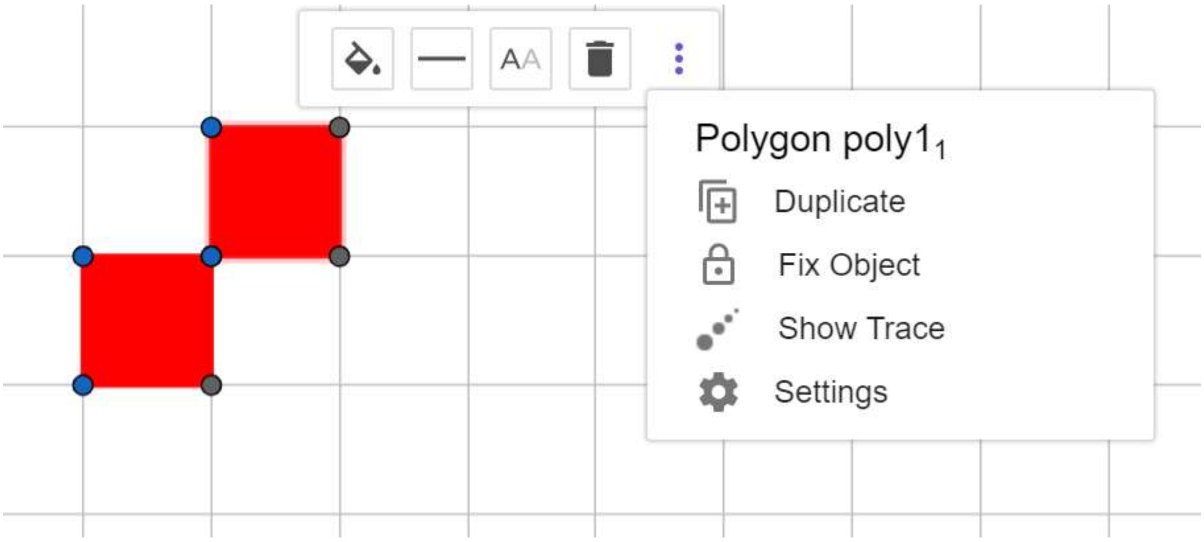
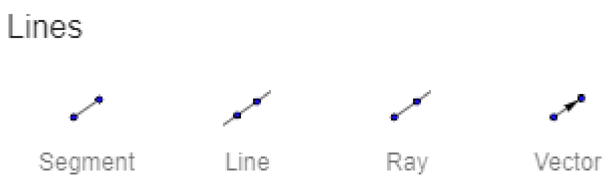


Figure 28. Duplication

4. **The main element - the cross, is a combination of 5 red squares. Our task is to find a possibility to copy this pattern element (cross), not to create a new cross each time. For that, there is a special tool – parallel shift (see Steps 1−3)**

For parallel shift:

Step 1. Create a vector[12] (we use it to make the shift)



Activate the transformation key

[12] A vector is a directed line segment, which has length and direction.

Step 2. Choose the object (part of the pattern)

Step 3. Click on the vector

Let's go through the steps:

1. Determine a vector

2. Choose: Translate by Vector, mark the whole cross and click on the vector. The drawing is cloned according to the direction and length of the vector.

3. Draw the rest of the vectors and clone the remaining elements of the pattern.

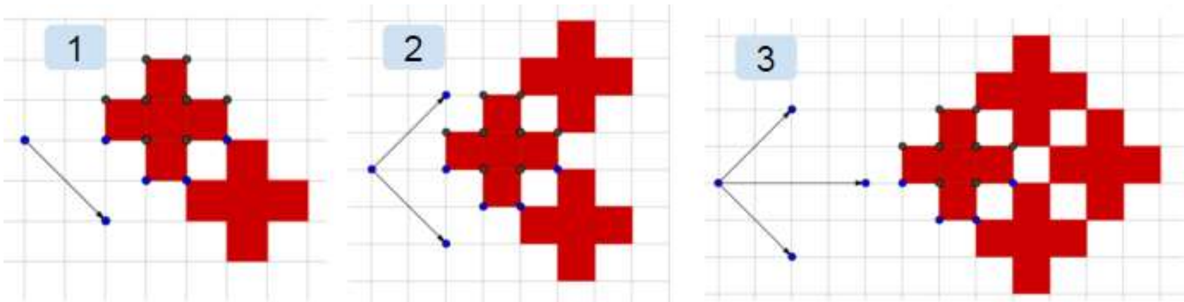


Figure 29. The cross (http://archive.novator.team/attach/42747)

4. Create the next element of the pattern.

   Determine a vector, which is the diagonal of the unit square (Figure 30).

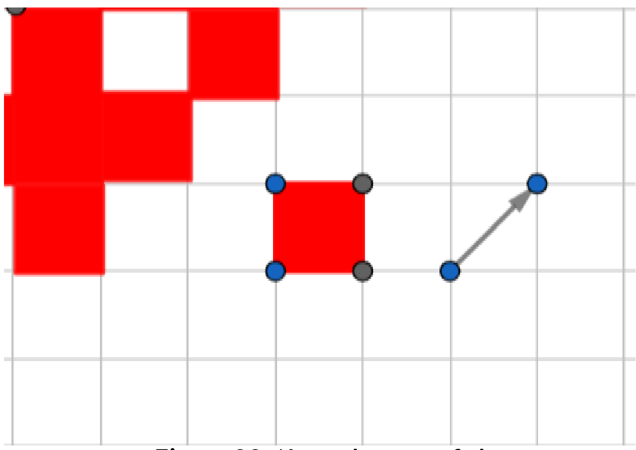


Figure 30. Next element of the pattern

Now create a composition that consists of seven squares positioned diagonally (Figure 31).

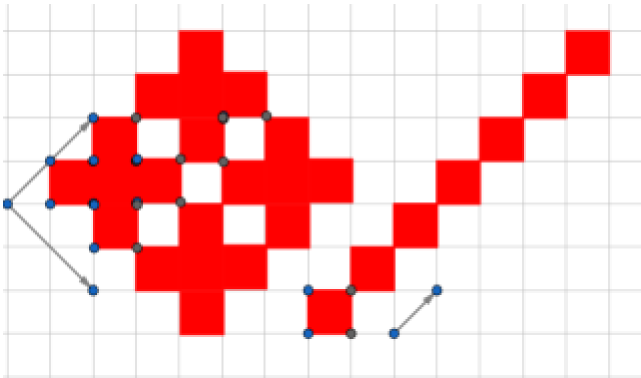


Figure 31. Diagonal of seven squeres

5. Hide the unnecessary elements. (Figure 32)

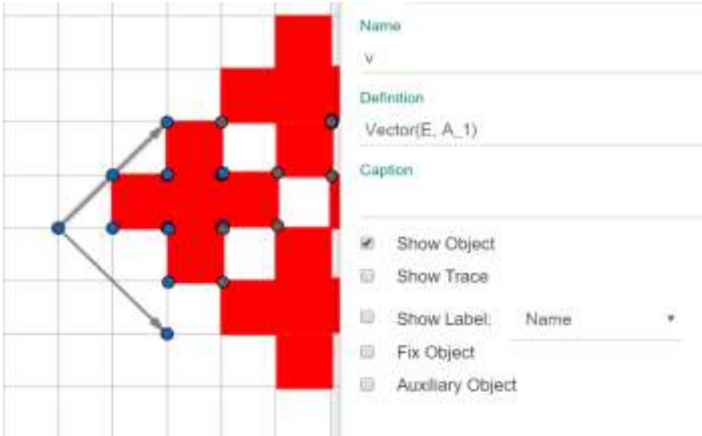


Figure 32. Show - Hide object

6. The last step is to reflect the combination over the line (Figure 33) to get the bigger repetition of the pattern. This repetition of the pattern starts to repeat on the belt band.

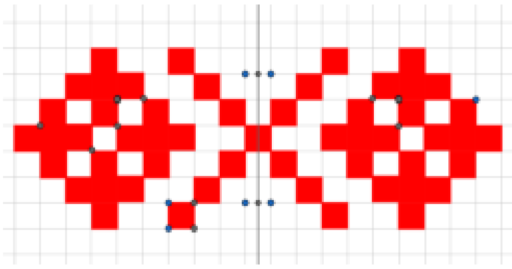


Figure 33. Reflect

7. With the help of a new vector, we clone the drawing to the right.

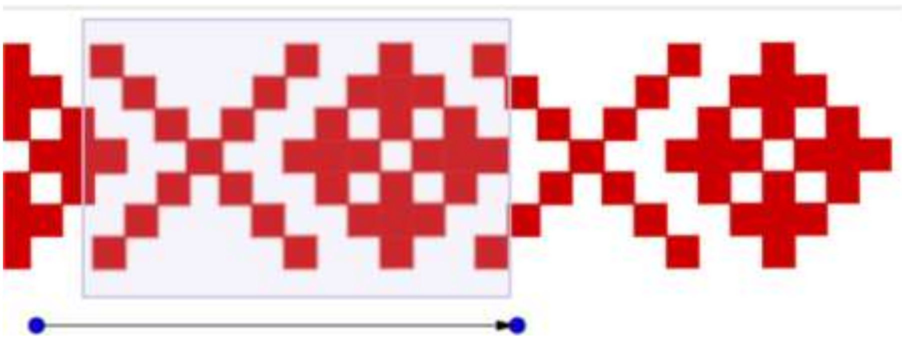


Figure 13. Shift to the right (http://archive.novator.team/attach/42751)

8. Colour the belt using the traditional colours of the Estonian belt band (Part I, tasks 3 and 4)

9. Design with a belt band with a pattern of your own.

Proposed exercises from Belgium

1. Architecture and Art

Pupils will learn more about the link between art, maths and architecture

In the year 2020 we will celebrate the *Ghent Altarpiece*, *the Adoration of the Mystic Lamb (Het Lam Gods*).
Considering this special occasion, we will give a visual tour through Gent looking for symmetry in Ghent's icon building.

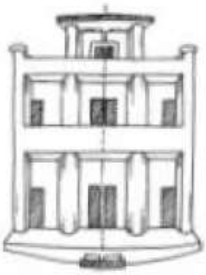
Link to Art

The Ghent Altarpiece, *the Adoration of the Mystic Lamb (Het Lam Gods*) is a very large and complex 15th-century polyptych altarpiece in St Bavo's Cathedral, Ghent, Belgium. It was begun c. the mid-1420s and completed before 1432 and is attributed to the Early Netherlandish painters and brothers Hubert and Jan van Eyck. The altarpiece is considered a masterpiece of European art and one of the world's treasures.

It is said that the Ghent Altarpiece is centred on an octangle.

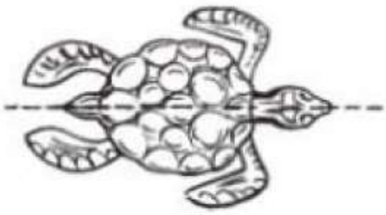
Link to Science

Symmetry is an important geometrical concept, commonly exhibited in nature and is used almost in every field of activity. Artists, professionals, designers of clothing or jewellery, car manufacturers, architects and many others make use of the idea of symmetry. The beehives, the flowers, the tree leaves, religious symbols, rugs, and handkerchiefs – everywhere you find symmetrical designs.



Architecture

Engineering

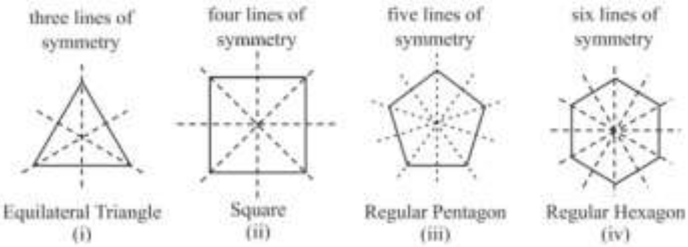


Nature

Link to Mathematics

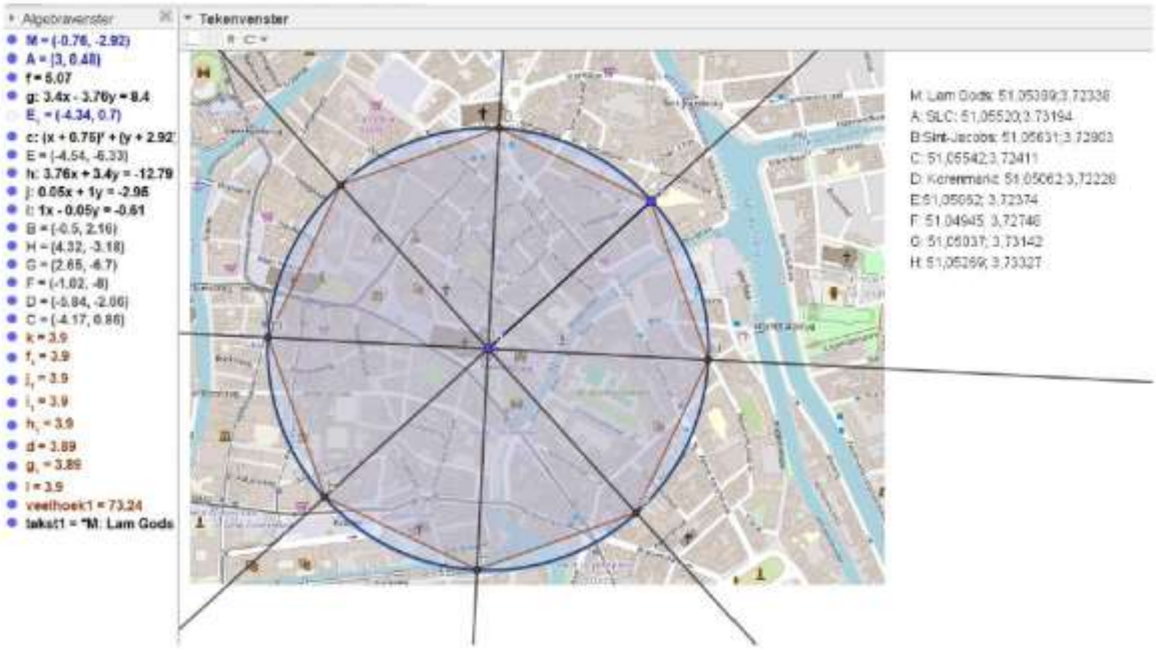
A polygon is a closed figure made of several line segments. The polygon made up of the least number of line segments is the triangle. A polygon is said to be regular if all its sides are of equal length and all its angles are of equal measure. Thus, an equilateral triangle is a regular polygon of three sides.

The regular polygons are symmetrical figures and hence their lines of symmetry are quite interesting. Each regular polygon has as many lines of symmetry as it has sides. We say, they have multiple lines of symmetry.



Game element

During the visual tour, pupils will look for symmetry in Ghent's icon buildings. There will be eight locations to visit. When answering questions correctly pupils will receive a coordinate.
These eight coordinates will form an octangle, in which the centre of the lines of symmetry will give you the location of the St Bavo's Cathedral, home of *the Ghent Altarpiece*.



Link to ICT

Since this is an international programme, the tour through Ghent will be visual. We will use different digital sources for this project.

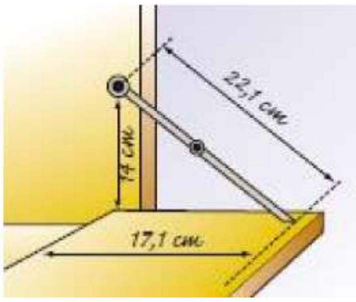
2. Pythagoras

Pupils will learn more about the link between art, maths and architecture.

Pythagorean Theorem has lots of applications in real life.

Even the Egyptians knew how to use this to build their pyramids.

In construction it is often used to form a rectangle.

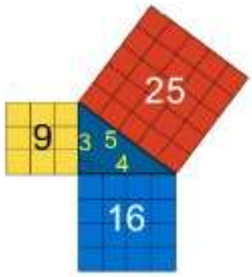


Link to Mathematics

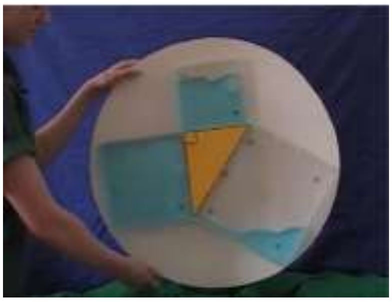
In mathematics, the Pythagorean Theorem, also known as Pythagoras' Theorem, is a fundamental relation in Euclidean geometry among the tree sides of a right triangle. It states that the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides. The theorem can be written as an equation relating the lengths of the sides a, b and c, often called the Pythagorean equation:

$a^2+b^2 = c^2$

where c represents the length of the hypotenuse and a and b the lengths of the triangle's other two sides.

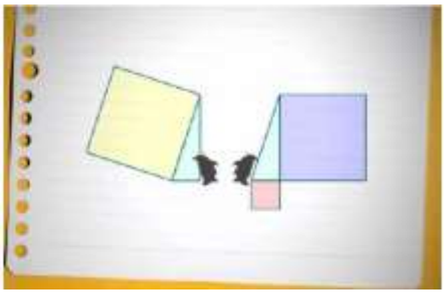


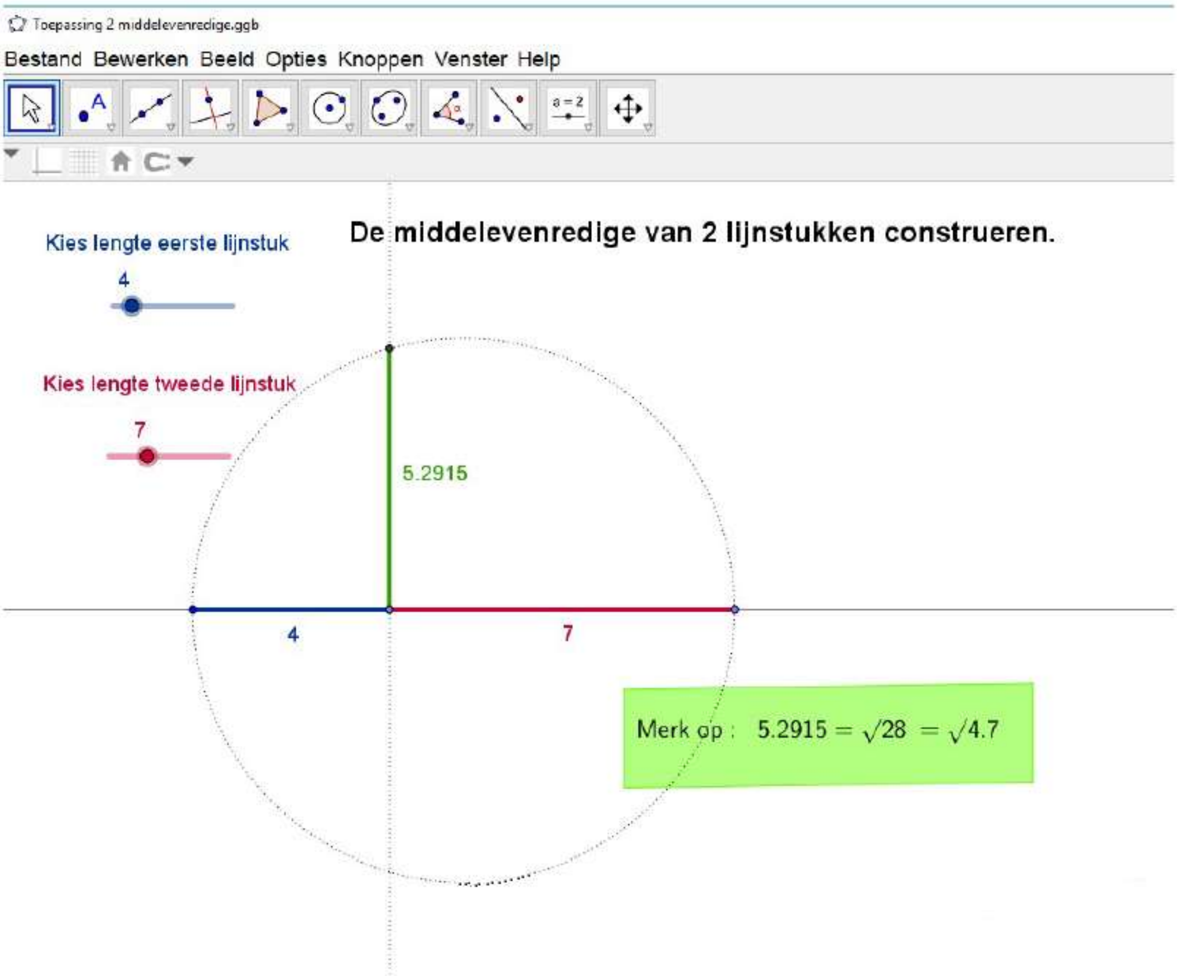
Link to ICT

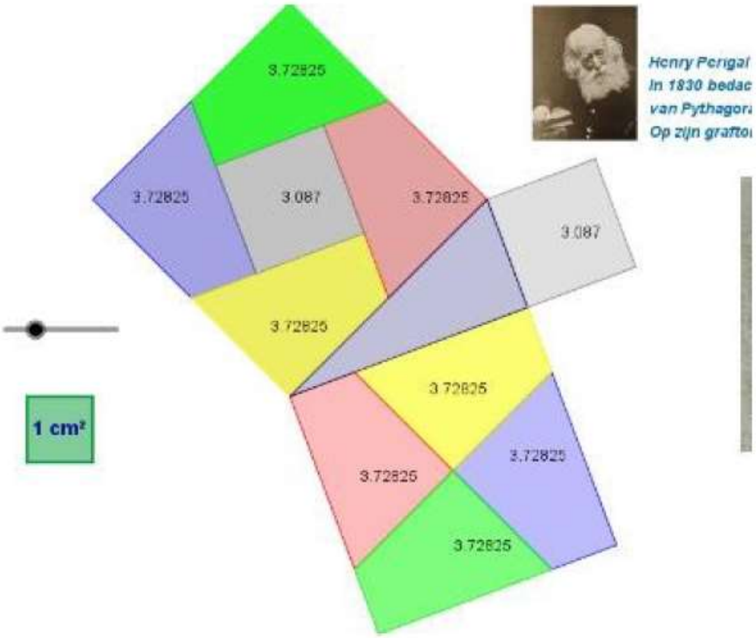


The pupils discover the different applications and ways to proof Pythagorean Theorem using a large range of applets and movies.

Using 3D models, pupils get insight in mathematical problems with 3D shapes.






3.72825
3.72825
3.087
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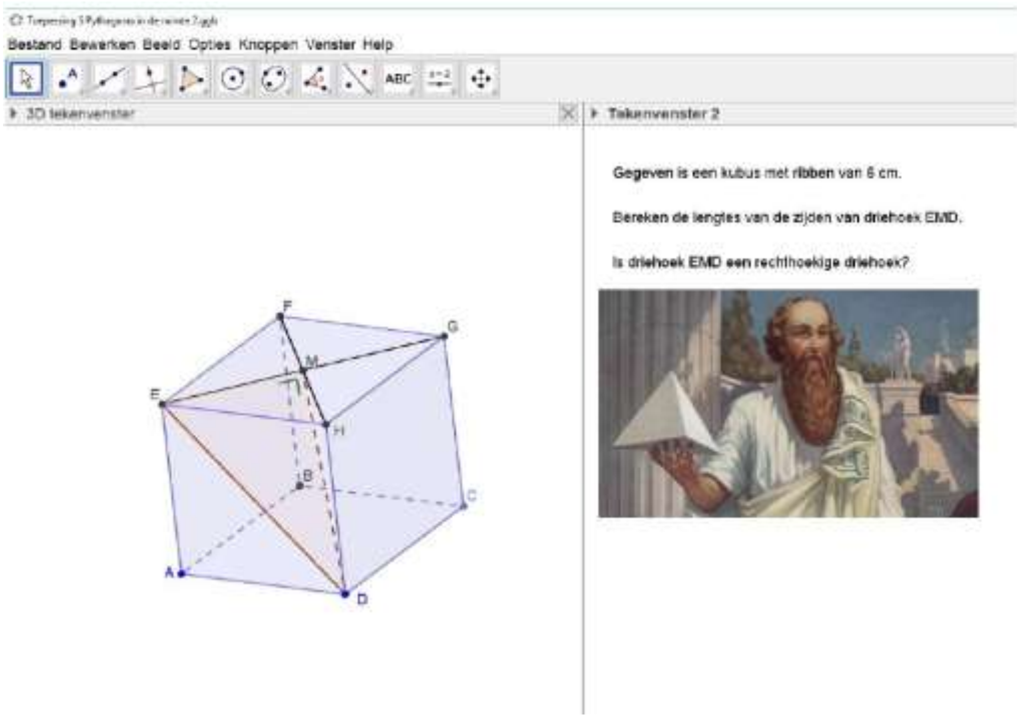
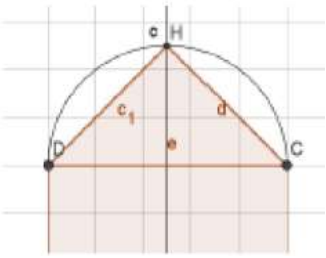
3.72825
1 cm²
Henry Perigal

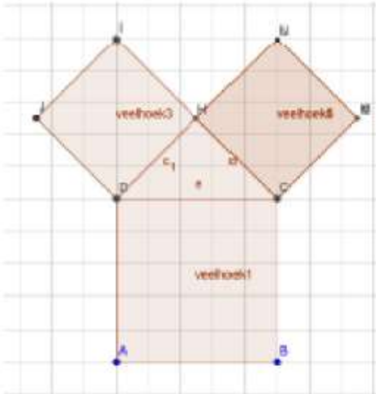

Bestand Bewerken Beeld Opties Knoppen Venster Help
3D tekenvenster
Tekenvenster 2
Gegeven is een kubus met ribben van 6 cm.
Bereken de lengtes van de zijden van driehoek EMD.
Is driehoek EMD een rechthoekige driehoek?
F
G
M
E
H
B
C
A
D

Link to Art

Pupils draw their Pythagoras tree using the free software GeoGebra.

They can choose to decorate it whatever way they want.





Game element

The pupils answer different questions about Pythagorean Theorem. Each correct answer will gain them a part of the Pythagoras tree at the end. The better they score, the nicer their tree. The group with the most beautiful tree wins.

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